



1. Find the solution of the following equation:

(a) $x^2 y'' - 2xy' + 2y = x^4 e^x$ (10%)

(b) $y'' - 4y' + 4y = (x+1)e^{2x}$ (10%)

(c) $y'' + 2y' - 4y = 1$; $y(0) = y'(0) = 0$ (10%)

2. A large tank is filled with 500 gallons of pure water. Brine containing 2 pounds of salt per gallon is pumped into the tank at a rate of 5 gal/min. The well-mixed solution is pumped out at the same rate. Find the number $A(t)$ of pounds of salt in the tank at time t . What is concentration of the solution in the tank at $t=5$ min? (10%)

3. Evaluate the surface integral $\iint_S G(x, y, z) ds$, where $G(x, y, z) = x$,

S : the portion of the cylinder $z = 2 - x^2$ in the first octant bounded by $x=0$, $y=0$, $y=4$, $z=0$. (10%)

4. (13%) Find the inverse of

$$A = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 2 & 1 & 1 & 5 \\ 0 & 0 & 2 & 3 \\ 4 & 2 & 1 & 0 \end{bmatrix}.$$

5. (12%) Determine both the row rank and the column rank of

$$B = \begin{bmatrix} 1 & 2 & 3 & 4 & 1 \\ 2 & 0 & 1 & 6 & 0 \\ 3 & 4 & 5 & 1 & 2 \end{bmatrix}.$$

6. (13%) Find the Fourier transform of $5u(t)$, where $u(t)$ is the unit step function. Show your derivation clearly.

7. (12%) Suppose $x(t)$ is a periodic function with a period π , and within $[0, \pi]$ it is defined as

$$x(t) = 3e^{-t/2}, \quad 0 \leq t \leq \pi.$$

Determine the Fourier series of $x(t)$.