國立雲林科技大學
97學年度碩士班入學招生考試試題

系所：資工所
科目：離散數學

1. For each of the following Boolean expressions, answer the three questions: decide if it is (i) valid, (ii) satisfiable, (iii) unsatisfiable. (Give all applicable properties, with justifications.) (15 points)
   (a) $A \wedge \neg A \wedge \neg B$. Is it valid? Is it satisfiable? Is it unsatisfiable? (5 points)
   (b) $(A \Rightarrow B) \wedge (B \Rightarrow C) \wedge (C \Rightarrow \neg A)$. Is it valid? Is it satisfiable? Is it unsatisfiable? (5 points)
   (c) $(A \Rightarrow B) \vee (B \Rightarrow A)$. Is it valid? Is it satisfiable? Is it unsatisfiable? (5 points)

2. Given a binary tree t, the depth depth(f, t) of a leaf f in t is defined to be the length of the path from f to the root of f. (Hence, if t is just an atom f, then depth(f, t)=0). (25 points)
   (a) Let children(r, A, children(n, B, C)) represent a tree t with root r having A as its left child and the node n as its right child, and n has left child B and right child C. In this tree t, what are the depths of the 3 leaves A, B, C. (5 points)

   (b) If a leaf f is a leaf of tree t_1 and depth(f, t_1)=d, and t_2 is some other tree, what is the value of depth(f, children(r_1, t_1, t_2))? (You do not need to prove your answer.) (5 points)

   (c) Let us define

   $$L(t) = \sum_{f \mid f \text{ is a leaf of } t} \frac{}{2^{d(f)}}$$

   Verify that L(t) = 1 for the tree t in (a). (5 points)
   (d) Using tree induction, show that L(t)=1 for every binary tree t. (You may use the result of part (2b) as a lemma here without giving the proof.) (10 points)

3. A map is a set of n countries $C_1, \ldots, C_n$, plus a specification of which countries C_i are adjacent to which countries C_j. A feasible 2-coloring assigns one of two colors to each country, such that no adjacent countries have the same color. (For example, the squares of a chessboard have a feasible 2-coloring.) Given a map, explain how to construct a CNF expression that is satisfiable iff a feasible 2-coloring exists for the map. (10 points)

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4. (10%) Prove that for all $n \in Z^+$, $n > 3 \Rightarrow 2^n < n!$

5. (10%) For every positive integer n, show that $\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\cdots=\binom{n}{1}+\binom{n}{3}+\binom{n}{5}+\cdots$

6. (10%) Allen writes the consecutive integers 1, 2, 3, ... , n on a blackboard. Then Barbara erases one of these integers. If the average of the remaining integers is $35\frac{7}{17}$, what is n and what integer was erased?

7. (10%) With A = $\{x, y, z\}$, let f, g:A→A be given by $f=\{(x, y), (y, z), (z, x)\}$, $g=\{(x, y), (y, x), (z, z)\}$. Determine each of the following: $g \circ f$, f^{-1}, g^{-1}, $(g \circ f)^{-1}$, $(f^{-1} \circ g^{-1})$

8. (10%) Let A=$\{1,2,3,4,5\}\times\{1,2,3,4,5\}$, and define $\Re$ on A by $(x_1, y_1)\Re(x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$.
   (a) Determine the equivalence classes $[(1, 3)]$, $[(2, 4)]$, $[(1, 1)]$.
   (b) Determine the partition of A induced by $\Re$.