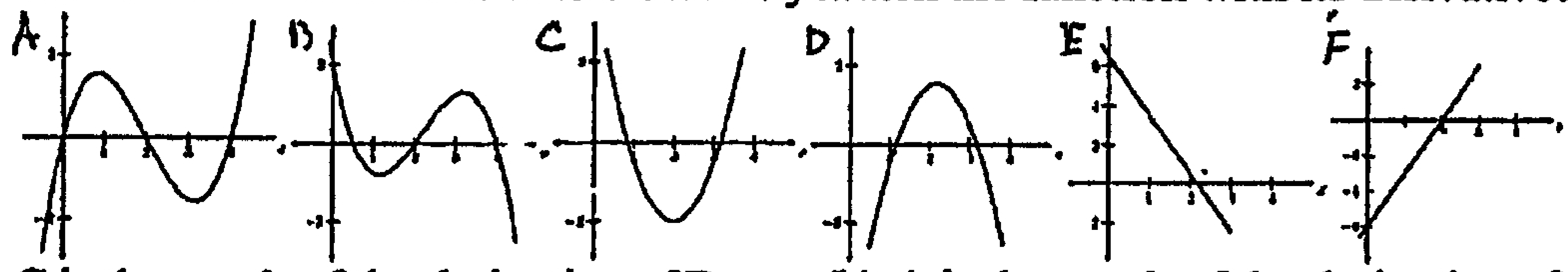
國立雲林科技大學
97學年度碩士班入學招生考試試題

系所：工管所、運籌所
科目：微積分

注意：請按照題號及子題號順序作答；不按題號順序作答不以計分。
第一題到第八題每題 5 分；第九題到第十八題每題 6 分。

1. Consider the function $y = x^4 - 2x^3$ for x between -1 and 3. Which one of the following statements is true?
   (a) There is a global maximum at $(\frac{3}{2}, -\frac{27}{16})$.
   (b) There is a local maximum at $(3,27)$.
   (c) There is a stationary point of inflection at $(0,0)$.
   (d) There is a global minimum at $(-1,3)$.
   (e) There is a local minimum at $(0,0)$.

2. Which of the statements below correctly match the function with its derivative?



(a) C is the graph of the derivative of B
F is the graph of the derivative of C
D is the graph of the derivative of A
E is the graph of the derivative of D.

(b) A is the graph of the derivative of C
C is the graph of the derivative of F
B is the graph of the derivative of D
D is the graph of the derivative of E

(c) C is the graph of the derivative of A
E is the graph of the derivative of C
D is the graph of the derivative of B
F is the graph of the derivative of D.

(d) C is the graph of the derivative of A
F is the graph of the derivative of C
D is the graph of the derivative of B
E is the graph of the derivative of D

3. A function f is decreasing for $x \geq 2$ and $f(2) = 20$, $f'(2) = -2$ and $f''(x) > 0$ for $x \geq 2$. Which of the following is a possible value for $f(4)$?
   (a) $f(4) = 16$ (b) $f(4) = 18$ (c) $f(4) = 20$ (d) $f(4) = 22$ (e) $f(4) = 24$

4. Which one of the following statements is correct?
   (a) if $f(t) = 5^t$ then $f'(t) = \frac{1}{\ln 5} 5^t$.
   (b) if $f(x) = x^{-3}$ then $f'(x) = -3x^{-2}$.
   (c) if $f(z) = \frac{1}{z} - \frac{2}{z^2}$ then $f'(z) = 1 - \frac{1}{z}$.
   (d) if $y = x^3 + 3x^2 + 5$ then $\frac{dy}{dt} = 3x^2 + 6x + 5$.
   (e) if $y = \frac{1}{\sqrt{t}}$ then $\frac{dy}{dt} = \frac{-1}{2\sqrt{t^3}}$.

5. Suppose $f(x,y) = x^3 e^{xy}$ Which one of the following statements is correct?
   (a) $\frac{\partial f}{\partial x} = 3x^2 e^{xy} + x^3 y e^{xy}$ and $\frac{\partial f}{\partial y} = x^4 e^{xy}$.
   (b) $\frac{\partial f}{\partial x} = 3x^3 y e^{xy}$ and $\frac{\partial f}{\partial y} = 3x^3 e^{xy}$.
   (c) $\frac{\partial f}{\partial x} = 3x^2 e^{xy} + x^4 e^{xy}$ and $\frac{\partial f}{\partial y} = x^3 y e^{xy}$.
   (d) $\frac{\partial f}{\partial x} = 3x^2 e^{xy}$ and $\frac{\partial f}{\partial y} = x^3 e^{xy}$.
   (e) $\frac{\partial f}{\partial x} = 3x^2 e^{xy} + x^2 y e^{xy}$ and $\frac{\partial f}{\partial y} = x^4 e^{xy}$.

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6. Which one of the following has not been differentiated correctly?

   (a) if $f(t)=e^{x^2+5}$ then $f'(t)=2xe^{x^2+5}$.

   (b) if $h(x)=\sqrt{x^2+5}$ then $h'(x)=\frac{x}{\sqrt{x^2+5}}$.

   (c) if $f(z)=\frac{1}{(z^3+2z+1)^2}$ then $f'(z)=\frac{3z^2+2}{(z^3+2z+1)^3}$.

   (d) if $h(x)=(2x^4+e^x)^3$ then $h'(x)=3(8x^3+e^x)(2x^4+e^x)^2$.

   (e) if $y=\sqrt[3]{(2x^2+3x+1)^2}$ then $\frac{dy}{dx}=\frac{2(2x+3)}{3\sqrt[3]{2x^2+3x+1}}$.

7. Which one of the following statements is correct?

   (a) $\lim\limits_{x\to\infty}\frac{x^2+e^x}{x+e^x}=\infty$ (b) $\lim\limits_{x\to 1}\frac{1-x}{e^x-e}=-e^{-1}$ (c) $\lim\limits_{x\to 0}\frac{\sin x^2}{x}=1$ (d) $\lim\limits_{x\to 1^-}\frac{\sqrt{1-x^2}}{x-1}=-1$ (e) $\lim\limits_{x\to 1}\frac{\ln x}{x-1}=0$

8. Find $\lim\limits_{n\to\infty}(-1)^n(\frac{n+1}{n})$

   (a) 1 (b) -1 (c) ∞ (d) $-\infty$ (e) None of the above

9. Find an expression for the area from 5 to 7 under the curve $y=x^3$ as a limit.

   (a) $\lim\limits_{n\to\infty}\sum_{i=1}^{n}(5+\frac{3i}{n})^3\frac{4}{n}$.

   (b) $\lim\limits_{n\to\infty}\sum_{i=1}^{n}(5+\frac{4i}{n})^3\frac{1}{n}$.

   (c) $\lim\limits_{n\to\infty}\sum_{i=1}^{n}(5+\frac{5i}{n})^3\frac{3}{n}$.

   (d) $\lim\limits_{n\to\infty}\sum_{i=1}^{n}(5+\frac{2i}{n})^3\frac{2}{n}$

   (e) $\lim\limits_{n\to\infty}\sum_{i=1}^{n}(5+\frac{3i}{n})^3\frac{3}{n}$

10. Evaluate the definite integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{x^2\sin x}{5+x^6}dx$.

    (a) 0 (b) -2 (c) 0.1 (d) 1 (e) 3

11. Find (approximately) the area of the region bounded by the curves

$$y=4+x^2,\quad y=4+e^{-x^2}.$$

    (a) $S=1.01$ (b) $S=0.96$ (c) $S=0.98$ (d) $S=0.99$ (e) $S=0.97$

12. Evaluate the integral $\int_0^1 x^3e^{-x^4}dx$.

    (a) $\frac{1}{5}(1-e)$ (b) $\frac{1}{4}(e^{-1}-1)$ (c) $\frac{1}{4}(1-e^{-1})$ (d) $4(1-e^{-1})$ (e) $5(1-e^{-1})$

13. Find $\lim\limits_{x\to-\infty}x^3e^x$.

    (a) $-\infty$ (b) 0 (c) 1/3 (d) 3 (e) ∞

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14. Evaluate the definite integral $\int \sin^3 2x \cos 2x dx$.
 (a) $-\frac{1}{10}\cos^5 2x + \frac{1}{6}\cos^3 2x + C$.
 (b) $\frac{1}{10}\cos^5 2x - \frac{1}{6}\cos^3 2x + C$.
 (c) $-\frac{1}{10}\sin^5 2x - \frac{1}{6}\sin^3 2x + C$.
 (d) $-\frac{1}{10}\sin^5 2x + \frac{1}{6}\sin^3 2x + C$
 (e) $10\sin^5 2x + 6\sin^3 2x + C$

15. Find the solution of the differential equation $x + 4y^3\sqrt{x^2+1}\frac{dx}{dy} = 0$ that satisfies the initial condition $y(0) = 6$.
 (a) $y = \sqrt[4]{1296 + \sqrt{x^2+1}}$.
 (b) $y = \sqrt[4]{1297 - \sqrt{x^2-1}}$.
 (c) $y = \sqrt[4]{1297 + \sqrt{x^2+1}}$.
 (d) $y = \sqrt[4]{1296 - \sqrt{x^2+1}}$
 (e) $y = \sqrt[4]{1297 - \sqrt{x^2+1}}$

16. Determine whether the series is convergent or divergent. If it is convergent, find its sum. $\sum_{n=1}^{\infty}\frac{4^n + 7^n}{28^n}$
 (a) divergent (b) $\frac{7}{4}$ (c) $\frac{4}{7}$ (d) $\frac{1}{2}$ (e) $\frac{9}{28}$

17. Find the Maclaurin series of $f(x) = x\cos(2x)$.
 (a) $\sum_{n=0}^{\infty}\frac{(-1)^n 2^n x^{2n+1}}{(2n)!}$.
 (b) $\sum_{n=0}^{\infty}\frac{(-1)^n 2^{2n} x^{2n}}{(2n)!}$.
 (c) $\sum_{n=0}^{\infty}\frac{(-1)^{n+1} 2^n x^{2n+1}}{(2n)!}$
 (d) $\sum_{n=0}^{\infty}\frac{(-1)^n 2^{2n} x^{2n+1}}{(2n)!}$
 (e) $\sum_{n=0}^{\infty}\frac{(-1)^n 2^{2n} x^{2n+1}}{n!}$

18. Calculate the double integral $\iint_R \frac{xy^2}{x^2+1} dA$, $R = \{(x, y) \mid 0 \le x \le 4, -3 \le y \le 3\}$.
 (a) 7ln5 (b) 7ln17 (c) 9ln17 (d) 9ln5 (e) 6ln17