國立雲林科技大學
97學年度碩士班入學招生考試試題 甲組
系所：工管所
科目：作業研究

注意：請按照題號及子題號順序作答；不按題號順序作答不以計分。

I.是非題 (True (T) or False (F), 共十五分；答對一題得五分、答錯一題扣三分。)

(1) Each optimal solution of one LP is a basic feasible solution of this LP.

(2) While solving a maximization integer programming problem (in branch and bound method), we obtain a subproblem with integer optimal solution x' with bound $Z' \geq Z^*$ (Z^* is the function value of the current candidate solution). In such case, x' can be the new candidate solution.

(3) Dijkstra's method is designed only for shortest path problems with positive arcs.

II.問答題(共八十五分) (除特別聲明，計算過程請勿附上)

II.1 (十分)

Given the following Linear Programming problem:

$$\begin{aligned} \min\ & 2x_{11}+4x_{12}+9x_{13}+8x_{21}+2x_{22}+6x_{23}+9x_{31}+4x_{32}+16x_{33} \\ \text{s.t.}\ & x_{11}+x_{12}+x_{13}=1,\ x_{21}+x_{22}+x_{23}=1,\ x_{31}+x_{32}+x_{33}=1 \\ & x_{11}+x_{21}+x_{31}=1,\ x_{12}+x_{22}+x_{32}=1,\ x_{13}+x_{23}+x_{33}=1 \\ & x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33} \geq 0 \end{aligned}$$

Please use "Northwest Corner" Method to find an initial basic feasible solution for this problem.

(A). What are your basic variables and their corresponding values? (5分)

(B). What is the entering variable and its corresponding leaving variables? (5分)

II.2 (五分)

Given the following Linear Programming problem:

$$\begin{aligned} \max\ & 60x_1+30x_2+20x_3 \\ \text{s.t.}\ & 8x_1+6x_2+x_3 \leq 48 \\ & 4x_1+2x_2+1.5x_3 \leq 20. \\ & 2x_1+1.5x_2+0.5x_3 \leq 8 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

and its optimal solution $(x_1, x_2, x_3) = (2, 0, 8)$, what can be said (must be zero, must be nonzero or undetermined) about the optimal values of the dual variable (y_1, y_2, y_3) and dual excess variables (r_1, r_2, r_3) without considering its dual problem?

II.3 (二十分)

Given the following Linear Programming problem:

$$\begin{aligned}
\max\quad & x_1 - 5x_2 + 3x_3 + 9x_4 + x_5 - x_6 \\
\text{s.t.}\quad & x_1 - 2x_2 + x_3 + 2x_4 + 3x_6 = 6 \\
& x_1 + x_2 - x_4 + x_5 + 2x_6 = 3 \quad , \\
& x_1 - 2x_4 + 3x_6 = 3 \\
& x_1,\ x_2,\ x_3, x_4, x_5, x_6 \geq 0
\end{aligned}$$

and one intermediate step of the simplex tableau for above LP.

x_1	x_2	x_3	x_4	x_5	x_6	RHS
0	(a)	0	(b)	0	(c)	(d)
0	−2	1	4	0	0	3
0	1	0	1	1	−1	(e)
1	0	0	−2	0	3	3

(A). What are the values of (a) to (e) in above tableau? (10分)

(B). Is the current solution degenerate? Why? (3分)

(C). Is the current solution optimal? Why? (4分)

(D). Are there multiple optimal solutions? Why? (3分)

II.4 (三十分)

Peter has \$3 dollars in his pocket and goes to gamble at a casino. Each time he bets for \$1, if he wins, with probability p, he gains \$1, otherwise he loses \$1. Peter's policy is to end the gambling game when the amount of money in his pocket reaches \$5 of \$1. Let us propose three processes to describe Peter's gambling: $\{X_n, n=0,1,2,\ldots\}$, $\{Y_n, n=0,1,2,\ldots\}$ and $\{Z_n, n=0,1,2,\ldots\}$, where $X_n = 1$ if Peter wins at the n^{th} gambling and $X_n = 0$, otherwise; Y_n denotes the accumulated number of winning that peter made up till game n (including game n) and Z_n is the amount of money that Peter has in his pocket after game n. Please give a detailed explanations for your answers of the following questions, points are given only if detailed explanations are provided.

(A). Is $\{X_n, n=0,1,2,\ldots\}$ a Markov chain? (10分)

(B). Is $\{Y_n, n=0,1,2,\ldots\}$ a Markov chain? (10分)

(C). Is $\{Z_n, n=0,1,2,\ldots\}$ a Markov chain? (10分)

II.5 (二十分)

Referring to the last problem (II.4), if $p=0.6$, find the probability that when Peter quits the games, he has \$5 in his pocket. (本題為計算題，請附上詳細計算過程)