

淡江大學 97 學年度碩士班招生考試試題

系別：數學學系

B

科目：機 率 論

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1. (25%) A common test for AIDS is called the ELISA test (Enzyme-Linked Immunosorbent Assay). Among 1,000,000 people who are given the ELISA test, we can expect results similar to those given in the table.

	B ₁ : Carry AIDS Virus	B ₂ : Do Not Carry AIDS Virus	Totals
A ₁ : Test Positive	4,885	73,630	78,515
A ₂ : Test Negative	115	921,370	921,485
Totals	5,000	995,000	1,000,000

If one of these 1,000,000 people is selected randomly, find the following probabilities:

(a) $P(B_1)$, (b) $P(A_1)$, (c) $P(A_1|B_2)$, (d) $P(B_1|A_1)$, (e) In words, what do parts (c) and (d) say?

2. (15%) Let the random variable X have the p.m.f. $f(x) = \frac{(|x|+1)^2}{9}$, $x = -1, 0, 1$. Compute $E(X)$, $E(X^2)$, and $E(3X^2 - 2X + 4)$.

3. (20%) The random variable X has a gamma distribution, denoted by $X \sim \Gamma(\alpha, \theta)$, if its p.d.f. is defined by $f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-x/\theta}$, $0 \leq x < \infty$.

- (a) Find the moment-generating function of X .
(b) Show that $E(X) = \alpha\theta$ and $Var(X) = \alpha\theta^2$.

4. (10%) Let X_1 and X_2 be two independent random variables with respective means μ_1 and μ_2 and variances σ_1^2 and σ_2^2 . Show that the mean and the variance of $Y = X_1 X_2$ are $\mu_1 \mu_2$ and $\sigma_1^2 \sigma_2^2 + \mu_1^2 \sigma_2^2 + \mu_2^2 \sigma_1^2$, respectively.

5. (20%) Let the independent random variables X_1 and X_2 be $N(0, 1)$ and $\chi^2(\gamma)$, respectively. Let $Y = X_1 / \sqrt{X_2 / \gamma}$ and $Z = \sqrt{X_2 / \gamma}$.

- (a) Find the joint p.d.f. of Y and Z . (Hint: $\chi^2(\gamma) \equiv \Gamma(\gamma/2, 2)$)
(b) Determine the marginal p.d.f. of Y .

6. (10%) Let X equal the birth weight in grams of a baby born in the Taiwan. Assume that $E(X) = 3320$ and $Var(X) = 660^2$. Let \bar{X} sample mean of a random sample of size $n = 225$. Find $P(3233.76 \leq \bar{X} \leq 3406.24)$, approximately.