淡江大學97學年度碩士班招生考試試題

系別：統計學系　　　　科目：基礎數學(含微積分、線性代數)

准帶項目請打「V」	
V	簡單型計算機

本試題共 1 頁， 大題

下列所有考題皆為計算或證明題，皆須附帶寫出計算或證明過程，否則不予計分。

1. Determine convergence or divergence of the following two series (State briefly the theorems or methods of test you employ in your work)

   a) $\sum_{k=1}^{\infty} \frac{\sin k}{k^2}$ (5%)　　**b)** $\sum_{n=1}^{\infty} (-1)^n a_n$ $\left(其中\ a_n = \int_2^{2+2/n} \frac{1}{x} dx\right)$ (5%)

2. Evaluate the given limits, if possible

   a) $\lim_{x \to 0} \frac{x - \int_0^x \cos t \ dt}{x - \int_0^x e^{t^2} dt}$ (6%)　　**b)** $\lim_{n \to \infty} \left(\frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \cdots + \frac{1}{\sqrt{n}\sqrt{n+n}}\right)$ (10%)

3. Use Mean Value Theorem to prove that: $\frac{x-1}{x} < \ln x < x-1$, whenever $x > 1$. (8%)

4. Prove that the function $f(x) = x^3 + 3x + 1$ has exactly one real root. (7%)

5. **a)** Write the Maclaurin series of the function $f(x) = e^x$ and its radius of convergence. (5%) **b)** Propose a strategy to approximate the definite integral $\int_0^1 e^{x^2} dx$ (just state the approximation strategy, don't evaluate it). (5%)

6. Let $A = [a_{ij}]_{n \times n}$, $a_{ij} \in R$ be an $n \times n$ matrix, prove that: the system $A_{n \times n} \cdot X_{n \times 1} = 0_{n \times 1}$ has non-trivial solution (i.e. the system has solution X that is not zero vector) $\Leftrightarrow$ (if and only if) $\det(A) = 0$. (10%)

7. Let $A = \begin{pmatrix} 2 & t^2 & 5 \\ 0 & t+1 & t \\ 0 & t+5 & 2t+5 \end{pmatrix}$, $t \in R$. **a)** For $t = 0$, use Cramer's rule to calculate A^{-1}. (8%) **b)** For all $t \in R$, prove that A^{-1} always exists. (5%)

8. Let $T: R^3 \to R^4$ be a linear transformation, where for 3×1 vector X, $T(X) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} X$. Find the bases respectively for **a)** the kernel (null space) of T and **b)** the image of T. (10%)

9. Let $A = \begin{pmatrix} 3/5 & 4/5 \\ 2/5 & 1/5 \end{pmatrix}$. **a)** Find the eigenvalues of A and their corresponding eigenvectors (6%); **b)** prove that when $n \in N$, $A^n \to \begin{pmatrix} 2/3 & 2/3 \\ 1/3 & 1/3 \end{pmatrix}$, as $n \to \infty$ (where $A^2 = A \times A$, and $A^n = A^{n-1} \times A$ for $n = 3, 4, 5, \cdots$). (10%)