

銘傳大學 97 學年度研究所碩士班招生考試

應用統計資訊學系碩士班

機率論試題(第三節)

(第 1 頁共 1 頁)(限用答案本作答)

☐ 可使用計算機 ☒ 不可使用計算機

1. (20 points) Suppose that random variable X is uniformly distributed distribution over the interval $(0, 1)$; that is, $f_X(x) = I_{(0,1)}(x)$. Assume that the conditional distribution of Y given $X = x$ has a binomial distribution with parameters n and $p = x$; i.e.,

$$P[Y = y | X = x] = \binom{n}{y} x^y (1-x)^{n-y} \text{ for } y = 0, 1, \dots, n.$$

- (a) Find $E[Y]$.
(b) Find the distribution of Y .

2. (12 points) Let $f_{X,Y}(x, y) = e^{-(x+y)} I_{(0,\infty)}(x) I_{(0,\infty)}(y)$. Find $P[2 < X+Y < 3]$ and $P[X < Y | X < 3Y]$.

3. (12 points) Let X be a random variable with probability density function given by

$$f_X(x) = 1-x | I_{[0,1]}(x).$$

Find the mean and variance of X .

4. (12 points) Let X_1 , X_2 and X_3 be uncorrelated random variables with common variance σ^2 . Find the correlation coefficient between X_1+X_2 and X_2+X_3 .

5. (12 points) If X has a normal distribution with mean μ and variance σ^2 , find the distribution, mean, and variance of $Y = e^X$.

6. (12 points) Derive the mean of the $F_{m,n}$ distribution.

7. (20 points) Let $X_1, X_2, \dots, X_n$ be a random sample from the probability density function $f(x) = \frac{1}{\beta} e^{-x/\beta}, 0 < x < \infty$.

- (a) Find a function of $X_1, X_2, \dots, X_n$, $g(X_1, X_2, \dots, X_n)$, such that $E[E[g(X_1, X_2, \dots, X_n)]] = 1/\beta$.
(b) Find two unbiased estimators of β^2 .

試題完