銘傳大學 97 學年度研究所碩士班招生考試

電腦與通訊工程學系碩士班

離散數學試題(第二節)

(第 1 頁共 2 頁)(限用答案本作答)　　☑可使用計算機 ☐不可使用計算機

1. (30 points, 2 points for each) True/False Questions.

   (1) If the statement $p \rightarrow q$ is true, then q is true.

   (2) Let $P(x, y)$ be a propositional function. If $\forall x \exists y P(x, y)$ is true, then $\exists y \forall x P(x, y)$ is true.

   (3) The equation $12x + 9y = 16$ has an integral solution.

   (4) The function $f : \mathbf{Z} \times \mathbf{Z} \rightarrow \mathbf{Z}$ is onto if $f(m, n) = m - n$.

   (5) $\dfrac{1}{1 - x^2} = 1 + x^2 + x^4 + x^6 + x^8 + \cdots$ for $|x| < 1$

   (6) Let m be a positive integer and let a and b be integers. Then $(a + b) \bmod m = (a \bmod m) + (b \bmod m)$.

   (7) The relation R on a set A is transitive if and only if $R^n \subseteq R$ for $n = 1, 2, 3, \ldots$.

   (8) The relation $R = \{(a, b) | ab \geq 0\}$ is on the set of all integers. The relation R is a partial ordering.

   (9) Let R_1 and R_2 be two relations on a set A. If R_1 and R_2 are transitive, then $R_1 \cup R_2$ is transitive.

   (10) If row i of an incidence matrix with the ordering $v_1, v_2, \ldots, v_n$ contains k 1's, the vertex v_i has degree k.

   (11) An n-hypercube is a bipartite graph.

   (12) If a graph has a Hamilton cycle, then it has an Euler cycle.

   (13) Let K_n be a complete graph on n vertices. Then the graph K_6 is not planar.

   (14) The height of a full binary tree with 31 vertices is 4.

   (15) A Huffman tree is always balanced.

2. (10 points, 5 points for each) How many solutions are there to the equation

   $$x_1 + x_2 + x_3 = 17,$$

   where x_1, x_2, and x_3 are nonnegative integers such that

   (1) $x_1 \geq 1$, $x_2 \geq 2$, and $x_3 \geq 3$?

   (2) $x_1 \leq 5$?

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3. (10 points) Show that $\sqrt{2}$ is irrational.

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(第2頁共2頁)(限用答案本作答)　　☑可使用計算機 ☐不可使用計算機

4. (10 points) What is the solution of the recurrence relation

$$a_n = a_{n-1} + 2a_{n-2}$$

with $a_0 = 1$ and $a_1 = 7$?

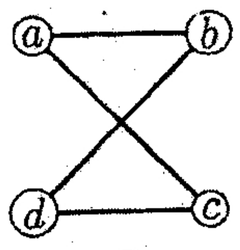
5. (10 points) Find all solutions to the system of congruences.

$$x \equiv 2(\mathbf{mod}\ 3)$$
$$x \equiv 3(\mathbf{mod}\ 5)$$
$$x \equiv 2(\mathbf{mod}\ 7)$$

6. (10 points) Show that $3x^2 + 8\log x$ is $\Theta(x^2)$.

7. (10 points) Prove that 3 divides $n^3 + 2n$ whenever n is a positive integer.

8. (10 points) How many paths of length four are there from a to d in the simple graph G?



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