

銘傳大學 97 學年度研究所碩士班招生考試
資訊工程學系碩士班與資訊傳播工程學系碩士班
資訊數學(含離散數學、線性代數)試題(第二節)

(第 1 頁共 2 頁)(限用答案本作答)

☐可使用計算機 ☒不可使用計算機

離散數學(50%)

1. Let $A = B = C = R$ (real numbers), and let $f: A \rightarrow B$, $g: B \rightarrow C$ be defined by $f(a) = a + 1$ and $g(b) = b^2 + 2$. Find (10%)
 - (a). $(g \circ f)(-2)$
 - (b). $(f \circ g)(-2)$
 - (c). $(g \circ f)(x)$
 - (d). $(f \circ g)(x)$
 - (e). $(f \circ f)(y)$
2. Let G be the set of all nonzero real numbers and let $a * b = (ab)/2$. Show that $(G, *)$ is an Abelian group. (10%)
3. Let $(S_1, *_1)$, $(S_2, *_2)$, and $(S_3, *_3)$ be semigroups, and $f: S_1 \rightarrow S_2$ and $g: S_2 \rightarrow S_3$ be homomorphisms. Prove that $g \circ f$ is a homomorphism from S_1 to S_3 . (5%)
4. Let $S = (1 \times 2 \text{ matrices}, \square)$, where $[x, y] \square [w, z] = [x+w, (y+z)/2]$. Determine which of the following properties hold for this structure. (15%)
 - (a). Closure
 - (b). Commutative
 - (c). Associative
5. Find an explicit formula for the sequence defined by $c_n = 3c_{n-1} - 2c_{n-2}$ with initial conditions $c_1 = 5$ and $c_2 = 3$. (10%)

線性代數(50%)

6. Let A be a matrix defined below

$$A = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}$$

Compute A^{-1000} .

(5%)

本試題兩面印刷

銘傳大學 97 學年度研究所碩士班招生考試
資訊工程學系碩士班與資訊傳播工程學系碩士班
資訊數學(含離散數學、線性代數)試題(第二節)

(第二頁共二頁)(限用答案本作答)

☐ 可使用計算機 ☒ 不可使用計算機

7. Let A be a matrix defined below

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

Find the matrices P and D such that $A = PDP^{-1}$. (10%)

8. Find the QR-decomposition of

$$A = \begin{bmatrix} -1 & 2 \\ 0 & -1 \\ 1 & 2 \end{bmatrix} \quad (10\%)$$

9. Let A be a matrix defined below

$$A = \begin{bmatrix} 1 & 3 & -2 & 0 \\ 2 & 6 & -5 & -2 \\ 2 & 6 & 0 & 8 \end{bmatrix}$$

(a). Find a basis for the nullspace of A . (6%)

(b). Find the rank and nullity of A . (4%)

10. Let $T: R^2 \rightarrow R^3$ be the linear transformation defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ 5x_1 - 6x_2 \\ 5x_1 + 6x_2 \end{bmatrix}$$

本試題兩面印刷

Find the matrix $[T]_{B',B}$ for the transformation T with respect to the bases $B = \{u_1, u_2\}$ for R^2 and $B' = \{u'_1, u'_2, u'_3\}$ for R^3 , where

$$u_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \quad u'_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad u'_2 = \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}, \quad \text{and} \quad u'_3 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}. \quad (15\%)$$

試題完