靜宜大學 97 學年度碩士班招生考試試題

系所：應用數學系　　科目：微分方程　　共 1 頁

1. (10) Please solve $xy^2y'+y^3 = x\cos x$. (Hint: by the change of variable $z = y^{1-n}$)
2. (10) Suppose $y_1(x)$ is a solution of $y''+p(x)y'+q(x)y = 0$. Please find $y_2(x)$ such that $c_1y_1(x)+c_2y_2(x)$ are the general solutions of $y''+p(x)y'+q(x)y = 0$.
3. (10) Suppose $y_1(x), y_2(x)$ are two solutions of $y''+xy'+e^x y = 0$. Please compute $y_1(x)y'_2(x)-y_2(x)y'_1(x) = ?$
4. (10) If the half-life of a radioactive substance is 20 days, how long will it take for 99 percent of the substance to decay?
5. (10) Please solve $y'' = 2y(y')^3$.
6. (10) Suppose $\sum_{n=0}^{\infty} a_n x^{n+m}$ is a solution of $2x^2y''+x(2x+1)y'-y = 0$. Please find $m = ?$
7. (10) Please find the general solutions of $y''-2y'+y = e^x$.
8. (10) Please find the general solutions of $\begin{cases} x' = y; \\ y' = z; \\ z' = x; \end{cases}$
9. (10) (1) Please find the critical points of $\begin{cases} \dfrac{dx}{dt} = y^2-5x+6; \\ \dfrac{dy}{dt} = x-y. \end{cases}$

   (10) (2) Please determine the type and stability of each critical point.