靜宜大學 97 學年度碩士班招生考試試題

系所：應用數學系　　　　科目：線性代數　　　　共 1 頁

(一)(共 40 分) $A=\begin{bmatrix}1 & -2 & 1\\ 2 & -3 & 5\\ 1 & 0 & 7\end{bmatrix}$

(1) solve Ax = b for $b=\begin{bmatrix}0\\4\\8\end{bmatrix}$.

(2) Find rank(A).

(3) Find the Null(A) = $\{x \in R^3 | Ax = 0\}$ and the basis(基底) of Null(A).

(4) Find the Range(A) = $\{y \in R^3 | \exists x \in R^3, Ax = y\}$ and the basis of Range(A).

(二)(共 40 分)

(5)Use the Gram-Schmidt process to generate an orthogonal(垂直) set from

$\{\begin{bmatrix}1\\1\\0\end{bmatrix},\begin{bmatrix}0\\2\\1\end{bmatrix},\begin{bmatrix}1\\1\\6\end{bmatrix}\}$

(6) Find determinant of A, $A=\begin{bmatrix}1 & 2 & -1 & 0\\ 0 & 1 & -1 & 1\\ 0 & -4 & 3 & 2\\ 3 & 1 & 1 & -1\end{bmatrix}$ (i.e., det(A)=?)

(7) Find the eigenvalues (固有值)and its corresponding eigenvectors (固有向量) of A for $A=\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}$.

(8) $T: R^2 \to R^3$ is a linear transformation defined by $T(\begin{bmatrix}x_1\\x_2\end{bmatrix})=\begin{bmatrix}x_1-x_2\\x_1+x_2\\x_2\end{bmatrix}$, please

find a matrix A such that T(x)=Ax.

(三)(共 20 分)

(8) If A is nonsingular and λ is an eigenvalue of A.

Prove that λ is nonzero and $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.

(9) Prove the following statement:

If A, B are both nxn nonsingular matrix, then AB is also nonsingular.