

靜宜大學九十七學年度碩士班招生考試試題

系所： 應用數學系 科目： 統計學

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1. (a) (10%) State the Central Limit Theorem.
 (b) (10%) Let $Y \sim \text{Binomial}(n, p)$. Show that Y is approximately normal with mean np and variance $np(1-p)$ as $n \rightarrow \infty$.
2. (a) (10%) Let $X \sim N(\theta, \sigma^2)$, and let g be a differentiable function satisfying $E|g'(X)| < \infty$. Prove that $E[g(X)(X - \theta)] = \sigma^2 E[g'(X)]$.
 (b) (10%) Let $X \sim N(\theta, \sigma^2)$. Use Stein's Lemma to find $E[X^3]$.
3. (10%) Let $X_1, X_2, \dots, X_n$ be iid exponential random variables with mean β . Define $Y_1, Y_2, \dots, Y_n$ as $Y_i = \begin{cases} 1, & X_i > \beta \\ 0, & X_i \leq \beta \end{cases}$. Find the distribution of $Z = \sum_{i=1}^n Y_i$.
4. Let X, Y be random variables with joint density function
$$f(x, y) = \begin{cases} 8xy, & 0 \leq x \leq 1, 0 \leq y \leq x \\ 0, & \text{otherwise.} \end{cases}$$

 (a) (10%) Find the marginal distributions of X and of Y .
 (b) (5%) Find the covariance of X and Y .
5. (10%) Draw the boxplot for the data: 3, 5, 1, 7, 8, 17, 9, 16, 6. Note that the 1st and 3rd quartiles based on the data are 5 and 9, respectively.
6. (10%) Suppose $X \sim \text{Uniform}(0, 1)$. Find the distribution of Y where $Y = -\lambda \log(X)$.
7. For a simple regression $y = \alpha + \beta x + \varepsilon$, answer the following questions according to the output shown below.
 (a) (5%) Test the hypothesis that $\beta = 0$ versus $\beta \neq 0$ at the significance level of 0.05.
 (b) (5%) Find a 90% confidence interval for β .
 (c) (5%) Find the coefficient of determination, R^2 .

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.34920	0.25333	1.378	0.24
x	1.92880	0.06248	----	6.56e-06

Residual standard error: 0.3133 on 4 degrees of freedom

R-Squared: _____

Analysis of Variance Table

Source	Df	SS	MS	F value	Pr(>F)
Regression	1	93.527	93.527	953.03	6.56e-06
Residuals	4	0.393	0.098		

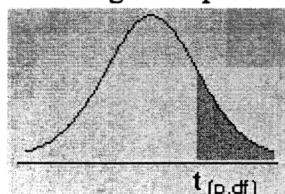
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t table with right tail probabilities



df \ p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869