中原大學97學年度碩士班入學考試

4月13日　16:00~17:30　應用數學系統計組
科目: 統計

誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你!

(共一頁，第一頁)

□可使用計算機，惟僅限不具可程式及多重記憶者　■不可使用計算機

1. (5pt) Please explain the simple random sampling.

2. (5pt) Please explain the Simpson's Paradox.

3. (5pt) Please explain the significance level.

4. (5pt) Please explain the simple linear regression model.

5. On the sample space $\{1, 2, 3\}$, let the probability be $P(i) = 1/3$, $i = 1, 2, 3$. Define the following r.v.'s
$$X_n(1) = X_n(2) = 1,\ X_n(3) = 0,\ n = 1, 2, 3, \ldots$$
and
$$X(1) = 0,\ X(2) = X(3) = 1.$$
Show that, as $n \to \infty$,
   (a) (5pt) X_n converges in distribution to X
   (b) (10pt) X_n does not converge in probability to X.

6. Let $X_1, \ldots, X_n$ be iid r.v.'s from $N(\mu, \sigma^2)$, $\bar{X} = \sum_{i=1}^n X_i/n$, and $S^2 = \sum_{i=1}^n (X_i - \bar{X})^2/(n-1)$.
   (a) (10pt) Show that $(\bar{X}, S^2)$ is sufficient for the parameters (μ, σ^2). Is S^2 alone sufficient for σ^2? estimator for μ?
   (b) (10pt) Assuming that σ^2 is a known constant, derive a (shortest) $1 - \alpha$ confidence interval for μ.

7. Let $X_1, \ldots, X_n$ be iid Bernoulli r.v.'s with successful probability $\theta \in (0, 1)$.
   (a) (10pt) What is the Cramér-Rao bound for this model?
   (b) (10pt) Now you are told that $n = 3$. Find the size-0.05 MP test for testing $H_0 : \theta = 0.05$ against $H_1 : \theta = 0.75$.
   (c) (5pt) What is the power of this test?

8. (10pt) Let $X_1, \ldots, X_n$ be iid r.v.'s from $U(\theta, 1)$, $\theta < 1$. Find the MLE for $\sin(\theta)$.

9. (10pt) Let X denote the length of time in seconds between two calls entering a college switchboard. Let m be the unique median of this contiuous-type distribution. We test the null hypothesis $H_0 : m = 6$ against the alternative hypothesis $H_1 = m < 6$. A random sample size 5 yield the following data:
$$9.5, 5.8, 5.1, 4.0, 1.7$$
Perform a sign test for this problem at significance level 0.05.