

國立臺北大學 100 學年度碩士班一般入學考試試題

系(所)別：統計學系

科目：基礎數學

第 1 頁 共 1 頁

☐可 ☒不可使用計算機

一、

(1). 求下列極限、微分或積分：

(a) $\lim_{x \rightarrow \infty} x - \sqrt{x^2 + 4x}$

(b) $\lim_{x \rightarrow 0} (\ln(1 - \cos 2x) - 2 \ln |x|)$

(c) $\int \frac{1}{\sqrt{(9-x^2)^3}} dx$

(d) $\cos(x^2 + y^2) = xye^y$, 求 dy/dx

(e) $\int \frac{\cos x}{2-3 \sin x} dx$

(f) $\int_0^2 \int_{y^2}^4 \sqrt{x} \cos x dx dy$ (30%)

(2). 令 $f(x, y) = \begin{cases} \frac{xy^3}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$:

(a) 請討論 $f(x, y)$ 在 $(0, 0)$ 是否連續。

(b) 求 $f_x(0, 0)$, 並討論 $f_x(x, y)$ 在 $(0, 0)$ 是否連續。

(c) 請討論 $f(x, y)$ 在 $(0, 0)$ 是否可微。(12%)

(3). 求 $F(x) = \int_0^{x^2} \sin t^2 dt$ 的 Maclaurin 展式, 及其收斂半徑及收斂區間。(8%)

二、

(1). Suppose that $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 5 & -1 \\ -3 & 6 & -1 \\ -3 & 5 & 0 \end{bmatrix}$. Are A, B similar? Are A, B row equivalent? Explain.

(8%)

(2). Suppose $v_1 = (1, 0, 0)$, $v_2 = (1, 0, -1)$. Find (a) the projection of $u = (1, 1, 1)$ onto the space spanned by v_1, v_2 and (b) the distance between u and the space. (Note: Please use the technique of linear algebra instead of the geometric method.) (10%)

(3). Please show that every orthonormal set is linear independent. (9%)

(4). Let $S = \{t^2 + 1, t + 1, t^2 + t + 1\}$, $T = \{t + 2, -t + 1\}$ be bases of P_2 and P_1 respectively. Suppose $L: P_2 \rightarrow P_1$ is a linear transformation and the matrix of L with respect to S and T is

$$\begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ -1 & -\frac{2}{3} & -\frac{4}{3} \end{bmatrix}. \text{ Please find } L(t^2 + 6t + 3). (10\%)$$

(5). Suppose $A = \begin{bmatrix} \frac{1}{3} & 0 & \frac{2\sqrt{2}}{3} \\ 0 & -1 & 0 \\ \frac{2\sqrt{2}}{3} & 0 & \frac{1}{3} \end{bmatrix}$.

(a). Please orthogonally diagonalize A . (8%)

(b). Please find $A^{21} + A^{20}$. (5%)

試題隨卷繳交