

國立台灣科技大學一百學年度碩士班招生試題

系所組別：工業管理系碩士班甲組、乙組、丙組

科目：統計學

(總分為100分)

(Total 100 Points.) There are 6 Problems in this exam. Show intermediate steps and formulas for partial credit. You must explain how you compute your results or answers for full credit.

1. (10 points) A random variable X has the following probability distribution, where a and b are constants:

$$f(x) = \begin{cases} ax^2 e^{-bx}, & 0 \leq x < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find the value of the constant a in terms of b . (5 points)

- (b) Find the probability distribution of Y , where $Y = \frac{mX^2}{2}$ and m is a constant. (5 points)

2. (20 points) Random variables X and Y has the following joint probability distribution:

$$f(x, y) = \begin{cases} \frac{1}{18} x^2 y, & 0 \leq x \leq 3, 0 \leq y \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find $P(X > 1, Y > 1)$. (4 points)

- (b) Find $P(1 < Y < 2.5)$. (4 points)

- (c) Find marginal probability distribution of the random variable X . (4 points)

- (d) Find $E(X)$. (4 points)

- (e) Find conditional probability distribution of Y given that $X = 1$. (4 points)

3. (20 points) The probability density function of the Erlang distribution is

$$f(x; k, \lambda) = \begin{cases} \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}, & x \geq 0, \lambda \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find the moment-generating function of the Erlang distribution. (7 points)

- (b) Find $E(X)$, the mean of the Erlang distribution. (6 points)

- (c) Find $Var(X)$, the variance of the Erlang distribution. (7 points)

4. (14 points) A random sample of 200 voters is selected and 120 are found to support a suit.

- (a) Find the 95% confidence interval for the fraction of the voting population favoring the suit. (7 points)

- (b) How large a sample is needed if we wish to be 95% confident that our sample proportion will be within 0.05 of the true fraction of the voting population? (7 points)

5. (21 points) A soft-drink machine at a steak house is regulated so that the amount of drink dispensed is approximately normally distributed with a mean of 200 milliliters and a standard deviation of 15 milliliters. The machine is checked periodically by taking a sample of 16 drinks and computing the average content. If $\bar{x}$ falls in the interval $192 < \bar{x} < 208$, the machine is thought to be operating satisfactorily; otherwise, we conclude that $\mu \neq 200$ milliliters.



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- (a) Find the probability of committing a type I error when $\mu = 200$ milliliters. (7 points)
- (b) Find the power of this test when $\mu = 210$ milliliters. (7 points)
- (c) Repeat to calculate the probability of committing a type I error when $\mu = 200$ milliliters if a sample of size 25 is taken. Use the same critical region. (7 points)
6. (15 points) In a shop study, a set of data was collected to determine whether or not the proportion of defectives produced by workers was the same for the day, evening, or night shift worked. The data were collected and shown in the following table. State the null and alternative hypotheses to be used in this test and draw appropriate conclusions with a 0.05 level of significance.

Shift	Day	Evening	Night
Defectives	40	55	70
Nondefectives	910	895	880

Note: χ_v^2 distribution with degrees of freedom v

$$P(\chi_v^2 > \chi_{\alpha, v}^2) = \alpha$$

Table Critical Values of the Chi-Squared Distribution

v	α			
	0.10	0.05	0.025	0.01
1	2.706	3.841	5.024	6.635
2	4.605	5.991	7.378	9.210
3	6.251	7.815	9.348	11.345
4	7.779	9.488	11.143	13.277
5	9.236	11.070	12.832	15.086
6	10.645	12.592	14.449	16.812
7	12.017	14.067	16.013	18.475
8	13.362	15.507	17.535	20.090
9	14.684	16.919	19.023	21.666
10	15.987	18.307	20.483	23.209



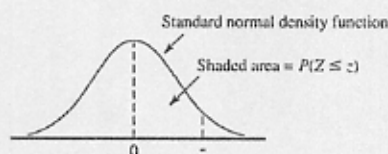
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Normal Probability Table:
Areas under the Normal Curve



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

