國立台灣科技大學一百學年度碩士班招生試題

系所組別： 自動化及控制研究所碩士班甲組、乙組

科　　目： 工程數學

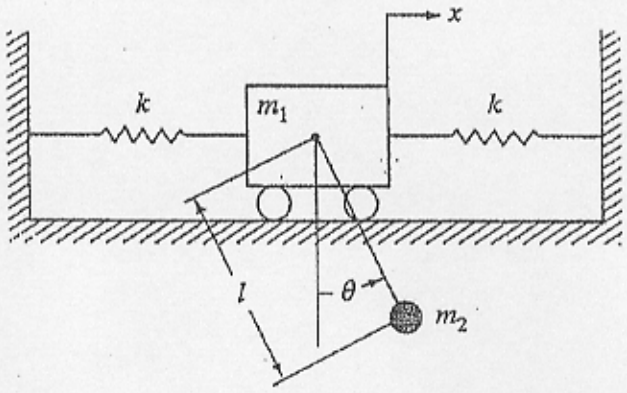
（總分為100分）

1. Solve the following differential equations:

(a) $2y''+3y'-2y=14x^2-4x-11;\quad y(0)=0, y'(0)=0.$ (10%)

(b) $x^2\dfrac{dy}{dx}=y-xy;\quad y(-1)=-1.$ (10%)

2. Consider the mechanical system shown in Problem 2. It is at rest for $t<0$. The pendulum m_2 is supported by mass m_1, which vibrates because of an elastic connection. The displacement x is measured from the equilibrium position for $t<0$. The angular displacement θ is measured from the vertical axis passing through the pivot of the pendulum. Assuming the initial conditions to be $x(0)=0.1$ m, $\dot{x}(0)=0$ m/s, $\theta(0)=0.1$ rad, and $\dot{\theta}(0)=0$ rad/s. Obtain the motion of the pendulum. Assume also that m_1=10kg, m_2=1kg, k=250N/m, and l=1m. (15%)



Problem 2

3. (a) Compute e^{At} for $\mathbf{A}=\begin{bmatrix}0 & 2 & -2\\ 0 & 1 & 0\\ 1 & -1 & 3\end{bmatrix}$. (5%)

(b)Use variation of parameters to solve the given system $\mathbf{X}'=\begin{pmatrix}3 & -1 & -1\\ 1 & 1 & -1\\ 1 & -1 & 1\end{pmatrix}\mathbf{X}+\begin{pmatrix}0\\ t\\ 2e^t\end{pmatrix}$. (10%)

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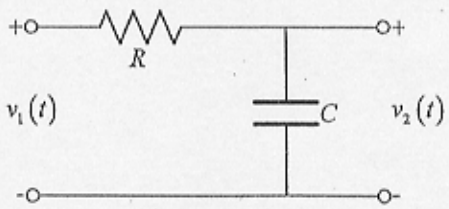
4. For the circuit shown in Problem 4 with $R=10k\Omega$ and $C=1\mu F$, determine

(a) The impulse response $h(t)$ of the system. (5%)

(b) The response $v_2(t)$ for $v_1(t)=\sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$, where

$$C_k=\begin{cases}1.5, & k=0\\ \dfrac{j3}{k\pi}, & k=1,\ 3,\ 5,\cdots\\ 0, & k=\pm 2,\ \pm 4,\cdots\end{cases} \quad \text{and} \quad C_{-k}=C_k^*. \ (5\%)$$

(c) The response $v_2(t)$ for $v_1(t)=10\cos\left(2500\pi t+30^\circ\right)$. (5%)

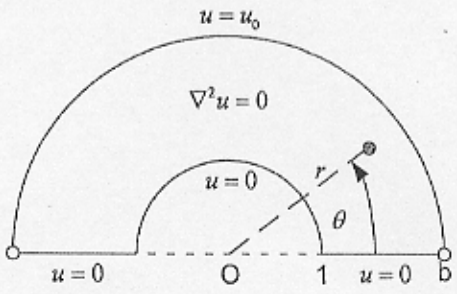


Problem 4

5. The boundary value problem in the semicircular plate shown in Problem 5 is

$$\nabla^2 u=\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0,\ 0<\theta<\pi,\ 1<r<b$$
$$u(1,\theta)=0,\ u(b,\theta)=u_0,\ 0<\theta<\pi$$
$$u(r,0)=0,\ u(r,\pi)=0,\quad 1<r<b.$$

Find the steady state temperature $u(r,\theta)$. (20%)



Problem 5

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6. The Laurent expansion of $f(z)=e^{(u/2)(z-1/z)}$ valid for $0<|z|$ can be shown to be $f(z)=\sum_{k=-\infty}^{\infty} J_k(u)z^k$, where $J_k(u)$ is the Bessel function of the first kind of order k. Use the contour $C:|z|=1$ and

$$a_k=\frac{1}{2\pi i}\oint_C \frac{f(s)}{(s-z_0)^{k+1}}\,ds,\ k=0,\ \pm1,\ \pm2,\cdots,$$

to show that the coefficients $J_k(u)$ are given by $J_k(u)=\frac{1}{2\pi}\int_0^{2\pi}\cos(kt-u\sin t)\,dt$. (15%)