國立臺北科技大學100學年度碩士班招生考試

系所組別：2300 資訊工程系碩士班

第二節　離散數學與演算法　試題

注意事項：

1. 本試題共十題，配分共100分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (10 pts) Solve the recurrence relation $a_n = 3a_{n-1} + 2^n$, $a_0 = 1$.

2. (10 pt) Let R and S be two relations on {1, 2, 3, 4} as follows:
R={(2,1), (2,3), (3,1), (3,4), (4,1), (4,3)},
S={(1,1), (1,3), (2,2), (3,1), (3,3), (4,4)}.
Please answer the following questions.
   (1) (4 pts) Determine if each relation is an equivalence relation or not. If yes, find the *equivalence classes* of the equivalence relation; if no, please describe your reasons why it's not an equivalence relation.
   (2) (6 pts) For relations that are not equivalence relation, please derive the directed graphs of the *reflexive closures*, *symmetric closures*, and *transitive closures* of the relations, respectively.

3. (8 pts) Please determine if each of the following statements is true or false.
   (1) (2 pts) If x and y are irrational, then xy is irrational.
   (2) (2 pts) If $a \equiv b$ (mod m) and $c \equiv d$ (mod m), where a, b, c, d, and m are integers with c and d positive and $m \geq 2$, then $ac \equiv bd$ (mod m).
   (3) (2 pts) There is a rational number x and an irrational number y such that xy is irrational.
   (4) (2 pts) If $2n$-1 is prime, then n is prime.

4. (10 pts) Let R and S be two relations. Mark by T(=true) or F(=false) each of the following:
   (1) (2 pts) If R and S are symmetric, then $R \cap S$ is symmetric.
   (2) (2 pts) If R and S are anti-symmetric, then $R \cup S$ is anti-symmetric.
   (3) (2 pts) If R is anti-symmetric, then R -1 is anti-symmetric, where R -1 is the inverse of R.
   (4) (2 pts) S is anti-symmetric if S is asymmetric and irreflexive.
   (5) (2 pts) If R and S are transitive, the $R \cup S$ is transitive.

5. (12 pts) Four applicants for a job are to be interviewed for 30 minutes each: 15 minutes with each of supervisors Nancy and Yolanda. (The interviews are in separate rooms, and interviewing starts at 9:00AM.)
   (1) (4 pts) In how many ways can these interviews be scheduled during a one-hour period?
   (2) (4 pts) One applicant, named Josephine, arrives at 9:00AM. What is the probability that she will have her two interviews one after the other?
   (3) (4 pts) Regina, another applicant, arrives at 9:00AM and hopes to be finished in time to leave by 9:50AM for another appointment. What is the probability that Regina will be able to leave on time?

6. (10 pts) In all of the five recurrences shown below, it is assumed that $T(1)=d$ for some constant d. State, using the "big oh" notation, the solution to each of the five recurrences shown below where c is a constant. Just state the answer -- **Don't** need to justify them.
   (1) (2 pts) $T(n)=2T(n/2)+c$
   (2) (2 pts) $T(n)=2T(n/2)+cn^2$
   (3) (2 pts) $T(n)=T(n/2)+cn$
   (4) (2 pts) $T(n)=T(n/2)+c \log n$
   (5) (2 pts) $T(n)=T(n/4)+T(3n/4)+cn$

7. (10 pts) Mark by T(=true) or F(=false) each of the following:
   (1) (2 pts) If problem P has an $\Omega(n^2)$ lower bound then, for any algorithm A that solves P, any instance of P that is given as input to A makes A take $\Omega(n^2)$ time.
   (2) (2 pts) If problem P has an $\Omega(n^2)$ lower bound then there can not exist an algorithm A that solves P in $O(n \log n)$ time.
   (3) (2 pts) If algorithm A solves problem P in $O(n \log n)$ time, then no instance of problem P can, when given to A as input, makes A take n^2 time.
   (4) (2 pts) If someone was able to give a polynomial time algorithm for a problem that is NP-complete, then this would imply that P is equal to NP.
   (5) (2 pts) Suppose problem P_1 can be reduced to problem P_2 in linear time. Then, if P_2 is NP-hard then \$ P_1 is NP-hard.

注意：背面尚有試題

8. (10 pts) Please answer each of the following problems shortly and concisely.
   (1) (5 pts) There is an iterative algorithm for finding the maximum and minimum which, thought not a divide-and-conquer based algorithm, is probably efficient. It works by comparing consecutive pairs of elements and then comparing the larger one with the current maximum and the smaller one with the current minimum. Write out the algorithm completely and analyze the number of comparisons it requires.
   (2) (5 pts) Let T be a binary tree rooted at r with vertex set V and edge set E. Suppose it is represented using adjacency list format. If node u is an ancestor of v, there is a path from r to v passing through u. Consider the function *ancestor*(u, v) which returns TRUE if u is an ancestor of v and FALSE otherwise. In order to have this function run in O(1) time, we are asked to design an algorithm to preprocess the tree. Please provide a linear time, i.e., $O(|V|+|E|)$ time, algorithm for this preprocess.

9. (10 pts) Recall the optimal binary search tree problem. Suppose that we are given four keys, a_1, a_2, a_3, and a_4, with the following access frequencies, where p_i is the frequency to access key a_i and q_i denotes the failure search between keys a_i and a_{i+1} with a_0=-∞ as well as a_5=∞.

i	0	1	2	3	4
p_i	×	5	2	4	3
q_i	3	2	3	4	2

Please derive an optimal binary tree for the given keys with access frequencies.

10. (10 pts) Consider the edge-weighted connected graph $G = (V, E)$ in Figure 1 where V is the vertex set and E is the edge set of G respectively.
   (1) (4pts) Please find the shortest path from vertex s to t by Dijkstra's algorithm. Show your work step by step.
   (2) (3pts) Write down the pseudo-code of Dijkstra's algorithm.
   (3) (3pts) What is the worst case running time of Dijkstra's algorithm? Note that, the running time will depend on what the priority queue is used.

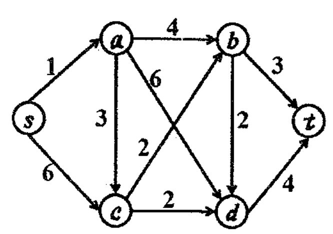

Figure 1: An edge-weighted connected graph