

科目	線性代數與機率	適用系所	通訊工程學系	時間	100分鐘
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※請務必在答案卷作答區內作答。

共2頁 第1頁

1. (15%) Assume that the jointly distribution function of X and Y is given by

$$f(x,y) = \begin{cases} xe^{-x(1+y)} & \text{if } 0 < x, y < \infty \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the conditional density $f(y | X = x)$?
 (b) Find the conditional expected value $E[Y | X = x]$?
 (c) Find the probability density function of the random variable $Z = X / Y$
2. (10%) Consider two boxes, the first box containing 4 black and 5 white balls, the second box containing 6 black and 2 white balls. A box is selected at random, and a ball is drawn at random from the selected box. What is the probability that the ball is black? What is the probability that the first box was the one selected, given that the ball is white?

3. (10%) The density function of the random variable X is given by

$$f(x) = \begin{cases} cx(1-x); & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find expected value of X , $E[X]$?
 (b) Find $E[X^2]$?
 (c) Find variance of X ; $\text{Var}[X]$?
4. (15%) If X is normal random variable with the parameter of mean $\mu_x = 0$ and variance

$$\sigma_x^2 = \sigma^2, \text{ that is the density function is given by } f(x) = \frac{1}{\sqrt{2\pi}} \exp^{-x^2/2\sigma^2}$$

Find the p.d.f. of the random variable $Y = e^X$?

5. Find conditions on a and b such that the system of linear equations

$$\begin{cases} x + 2y = 3 \\ ax + by = -9 \end{cases}$$

has

- (a) (5%) no solution.
 - (b) (5%) exactly one solution.
6. (8%) Let \mathbf{A} , \mathbf{B} , and $\mathbf{A} + \mathbf{B}$ be nonsingular matrices. Prove that $\mathbf{A}^{-1} + \mathbf{B}^{-1}$ is nonsingular by showing that $(\mathbf{A}^{-1} + \mathbf{B}^{-1})^{-1} = \mathbf{A}(\mathbf{A} + \mathbf{B})^{-1}\mathbf{B}$.
7. Please find the dimension of the subspace W shown as follows.
- (a) (8%) $W = \{[d \quad c - d \quad c] \mid c \in \mathbb{R}, d \in \mathbb{R}\}$.
 - (b) (8%) $W = \{[b \quad 3b \quad 0] \mid b \in \mathbb{R}\}$.

8. For a matrix $\mathbf{A} = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

- (a) (8%) Please find the eigenvalues of \mathbf{A} .
- (b) (8%) Please find the eigenvectors of \mathbf{A} .