

科目	作業研究	適用系所	工業工程與系統管理學系B組	時間	100 分鐘
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※請務必在答案卷作答區內作答。

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1. (15%) A fertilizer company purchases nitrates, phosphates, potash, and an inert chalk base at a cost of \$1,500, \$500, \$1,000, and \$100 per ton, respectively, and produces four fertilizers A , B , C , and D . The production cost, selling price, and composition of the four fertilizers are given below.

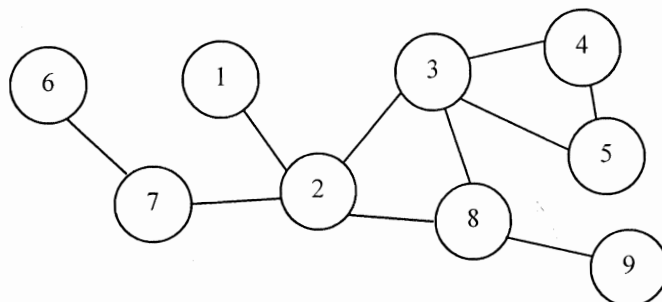
Fertilizer	Production cost (\$/ton)	Selling price (\$/ton)	Percentage composition by weight			
			Nitrates	Phosphates	Potash	Inert chalk base
A	100	350	5	10	5	80
B	150	550	5	15	10	70
C	200	450	10	20	10	60
D	250	700	15	5	15	65

During any week, no more than 1,000 tons of nitrate, 2,000 tons of phosphates, and 1,500 tons of potash will be available. The company is required to supply a minimum of 5,000 tons of fertilizer A and 4,000 tons of fertilizer D per week to its customers; but it is otherwise free to produce the fertilizers in any quantities it pleases. Formulate the problem of finding the quantity of each fertilizer to be produced by the company to maximize its profit.

2. (15%) The following is the current simplex tableau of a given maximization problem. The objective is to maximize $2x_1 - 4x_2$, and the slack variables are x_3 and x_4 . The constraints are of the \leq type.

	z	x_1	x_2	x_3	x_4	RHS
z	1	b	1	f	g	8
x_3	0	c	0	1	1/5	4
x_1	0	d	e	0	2	a

- (a). Find the unknowns a through g .
 (b). Is the tableau optimal?
3. (20%) The following network represents an electrical power distribution network connecting power generating points with power consuming points.



The arcs are undirected; that is, power may flow in either direction. Points 1, 4, 7, and 8 are generation points with generating capacities and unit costs given by the following table.

	Generating Point			
	1	4	7	8
Capacity (thousands of kilowatt hours)	100	60	80	150
Unit Cost (\$/1000 kilowatt hours)	50.0	63.5	75.0	84.5

Points 2, 5, 6, and 9 are consuming points with demands of 35,000 kwh, 50,000 kwh, 60,000 kwh, and 40,000 kwh, respectively. There is no upper bound on the transmission line capacity. The unit cost of transmission on each line segment is \$25.0 per 1,000 kwh. Please set up the power distribution problem as a network flow problem.

4. (20%) Consider the following LP model

$$\max z = 3x_1 + 2x_2$$

s.t.:

$$3x_1 + x_2 \leq 12$$

$$x_1 + x_2 \leq 6$$

$$5x_1 + 3x_2 \leq 27$$

$$x_1, x_2 \geq 0$$

- Construct the dual problem and solve this dual problem by using dual simplex method
- Use the complementary slackness theorem to prove that the solutions that you found are optimal solutions for both primal and dual problems.

5. (30%) It's a recession and people are waiting longer between haircuts to save a little money. Ms. H. R. Cut now finds her arrival rate reduced to 4/hr. Her service rate is still 6/hr and she still has only four chairs for waiting customers.

- How much of an eight-hour day is she not cutting hair?
- What is the average number waiting in the queue over all customers?
- What is the average time spent, by all customers, waiting in the queue?
- What fraction of customers does not have to wait at all?
- What is the probability that there are exactly 3 customers in the system?
- What is the probability that there are exactly 3 customers in the queue?