

東吳大學 100 學年度碩士班研究生招生考試試題

第 1 頁，共 1 頁

系級	財務工程與精算數學系碩士班 A、B 組	考試時間	100 分鐘
科目	微積分	本科總分	100 分

1. Let $\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$, $\Phi(x) = \int_{-\infty}^x \phi(z)dz$, and $\Phi^{-1}(x)$ is the inverse function of $\Phi(x)$.
 - (a) Find $\lim_{x \rightarrow \infty} \frac{1 - \Phi(x)}{\phi(x)}$. (10/100)
 - (b) Find $\frac{d}{dx} \Phi(\Phi^{-1}(x) + c)$, where c is a constant. (10/100)
 - (c) For $x > 0$, show that $\frac{1 - \Phi(x)}{\phi(x)} \leq \frac{1}{x}$. (10/100)
 - (d) Using the Taylor expansion, show that $\Phi(x) = \frac{1}{2} + \frac{1}{\sqrt{2\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)2^k k!}$ for $x \geq 0$. (10/100)
2. Let $g(x) = \max(x - c, 0)$, where c is a constant. Find $\int_0^{\infty} g(x)e^{-(x-c)} dx$. (10/100)
3. Show that $\int_1^{\infty} \frac{1 + e^{-x}}{x} dx$ is divergent. (10/100)
4. Find $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n+i}$. (10/100)
5. Find $\int_0^1 \int_{3y}^3 e^{x^2} dx dy$. (10/100)
6. Let $z^3 - yz - x = 0$. Show that $\frac{\partial z}{\partial x \partial y} = -\frac{3z^2 + y}{(3z^2 - y)^3}$. (10/100)
7. Explain why Newton's method fails when applied to the equation $x^{1/3} = 0$ with any initial approximation $x_1 \neq 0$. (10/100)