東吳大學100學年度碩士班研究生招生考試試題

系級	財務工程與精算數學系碩士班 A、B 組	考試時間	100 分鐘
科目	機率統計	本科總分	100 分

1) (5%) Let X be a random variable having density function $f(x)$ given by

$$f(x) = \begin{cases} 1/18 & x = 1, 3 \\ 16/18 & x = 2 \end{cases}$$

Calculate c such that

$$P(|X - E[X]| \geq c) = \frac{Var[X]}{c^2}$$

2) (10%) Let X and Y be two random variables having finite expectation. Suppose $P(X \geq Y) = 1$.

   (i) Show that $E[X] \geq E[Y]$.

   (ii) $E[X] = E[Y]$ if and only if $P(X = Y) = 1$.

3) (10%) Let $X_1, \cdots, X_n$ be independent random variables having a common density with mean μ and variance σ^2. Set $\bar{X} = \sum_{i=1}^{n} X_i/n$.

   (i) Show that

$$\sum_{i=1}^{n}(X_i - \bar{X})^2 = \sum_{i=1}^{n}(X_i - \mu)^2 - n(\bar{X} - \mu)^2$$

   (ii) Find

$$E\Big[\sum_{i=1}^{n}(X_i - \bar{X})^2\Big]$$

4) (20%) Let X be a uniformly distributed random variable over $(0, 1)$, and let Y be a uniformly distributed random variable over $(0, X)$. Find the joint density of X and Y and the marginal density of Y.

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5) (15%) Given Geometric density function

$$f(x) = p(1-p)^x,\ x = 0, 1, 2, \cdots,\ 0 < p < 1$$

Let X and Y be independent random variables each geometrically distributed with parameter p.

(i) Find the distribution of $\min(X, Y)$.

(ii) Find $P(Y \geq X)$.

(iii) Find the distribution of $X + Y$.

(iv) Let $M > 0$ be an integer. Compute the mean of $\min(X, M)$.

(v) Find $E[Y|X + Y = c]$ where c is a non–negative integer.

6) (20%) Suppose that the conditional density of Y given $\Lambda = \lambda$ is the Poisson density with parameter λ, i.e.,

$$f_{Y|\Lambda}(y|\lambda) = \frac{\lambda^y e^{-\lambda}}{y!},\ y = 0, 1, 2, \cdots$$

where the density of Λ is

$$f_\Lambda(\lambda) = \frac{\beta^\alpha \lambda^{\alpha-1} e^{-\lambda\beta}}{\Gamma(\alpha)},\ \lambda > 0$$

(i) Find the density of Y.

(ii) Find the density of Λ given $Y = y$.

7) (20%) Let the random variable X have a uniform distribution with density given by

$$f(x) = \frac{1}{2\sqrt{3}\sigma},\ \mu - \sqrt{3}\sigma \leq x \leq \mu + \sqrt{3}\sigma$$

where $-\infty < \mu < \infty$ and $\sigma > 0$. Find the maximum–likelihood estimates of μ and σ.