東吳大學100學年度碩士班研究生招生考試試題

系級	經濟學系碩士班	考試時間	100 分鐘
科目	統計學	本科總分	100 分

1. Assume that a random sample of size n is drawn from a population where $E(X_i)=\mu$, $Var(X_i)=\sigma^2$, $i=1,2,...,n$, and for $i\neq j$, X_i and X_j are independent.

(a) Prove the followings:

(i) $Var(X_i)=E(X_i^2)-[E(X_i)]^2$ (4%)

(ii) $E(\overline{X})=\mu$ and $Var(\overline{X})=\sigma^2/n$ (8%)

(b) Show that the sample variance s^2 is the unbiased estimator of σ^2; that is to prove $E(s^2)=\sigma^2$, where $\overline{X}=\sum_{i=1}^{n}\frac{X_i}{n}$, $s^2=\sum_{i=1}^{n}\frac{(X_i-\overline{X})^2}{n-1}$. (4%)

(Hint: use the results of (a) above)

(c) If μ is known, show that $\sum_{i=1}^{n}\frac{(X_i-\mu)^2}{n}$ is the unbiased estimator of σ^2. (4%)

(d) Show that $Var(aX_i+b)=a^2Var(X_i)$ where a,b are constants. (4%)

(Hint: use the definition of variance)

2. Assume that $Z=\frac{X-\mu}{\sigma}$ and $X\sim N(\mu,\sigma^2)$, show that $E(Z)=0$, $Var(Z)=1$. (Hint: use the result of question 1.(d) above) (4%)

3. Assume that the regression model is as follows:

$y_i=\beta x_i+u_i,\ \ i=1,2,...,n$

Where $u_i\ \underline{iid}\ N(0,\sigma^2)$ and x_i is a scalar (純量) and nonstochasitc, $\forall i=1,2,...,n$

(a) Derive the OLS(Ordinary Least Squares) estimator $\hat{\beta}^*$ of β. (6%)

(b) Show that $\hat{\beta}^*$ is the unbiased estimator of β. (6%)

4. If the $X_i's$ of Question 1 are as follows:

10 12 18 11 19

Find $\overline{X}$ and the unbiased estimate (s^2) of σ^2. (6%)

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5. A blood test is 99 percent effective in detecting a certain disease when the disease is present. However, the test also yields a false-positive result for 2 percent of the healthy patients tested. (That is, if a healthy person is tested then with probability 0.02 the test will say that this person has the disease.) Suppose 0.5 percent of the population has the disease. Find the conditional probability that a randomly tested individual actually has the disease given that his or her test result is positive(求出檢驗結果爲有病其真正有病之機率). (8%)

6. A lot of 200 items contains 40 defectives. Let X denote the number of defectives in a sample of 15 items. Determine the probability distribution of X for X=0,1,2,…15, if the sample is drawn one at a time
   (a) with replacement (抽出放回). (6%)
   (b) without replacement (抽出不放回). (6%)
   (c) Prove that $\sum_{i=1}^{15} p_r(X=i)=1$ in (a) above. (6%)

7. To test the hypothesis $H_0:\mu=100$ against $H_1:\mu\neq 100$ where μ is the population mean. A random sample of size 4 is chosen from normal population $N(\mu,\sigma^2)$. If the sample mean is $\overline{X}=95$, and $\alpha=5\%$, test H_0 for the following (b) and (c)
   (a) Explain that the distribution of $\overline{X}$ is $N(\mu,\sigma^2/4)$ (4%)
   (b) $\sigma^2=25$ where $t_{3,0.025}=3.182$, $Z_{0.025}=1.96$. (6%)
   (c) σ^2 is unknown, but the sample variance $s^2=36$. (6%)
   (d) find the 95% confidence interval for μ in (b), (c) above. (6%)
   (e) explain the p-value by using graphs (圖形) and words (文字) in (c) above. (6%)