注意事項:
1. 請以黑色或藍色筆作答。
2. 請依題號次序作答, 並標明題號, 否則不予計分。

Part I: Multiple Choice (25 points)
Only one answer is correct. Each question counts 2.5 points.

1. Two events A and B are said to be independent if:
(A) $P(A \text{ and } B) = P(A) \cdot P(B)$ (B) $P(A \text{ and } B) = P(A)+P(B)$ (C) $P(A/B) = P(B)$ (D) $P(B/A) = P(A)$

2. If X and Y are any random variables with $E(X) = 50$, $E(Y) = 6$, $E(XY) = 21$, $V(X) = 9$ and $V(Y) = 10$, then the relationship between X and Y is a :
(A) strong positive relationship (B) strong negative relationship (C) weak positive relationship (D) weak negative relationship

3. What proportion of the data from a normal distribution is within three standard deviations from the mean?
(A) 0.3413 (B) 0.4772 (C) 0.6826 (D) 0.9544

4. A sample of 250 observations will be selected at random from an infinite population. Given that the population proportion is 0.25, the standard error of the sampling distribution of the sample proportion is :
(A) 0.0274 (B) 0.50 (C) 0.0316 (D) 0.0548

5. Two samples are selected at random from two independent normally distributed populations. Sample 1 has 49 observations and has a mean of 10 and a standard deviation of 5. Sample 2 has 36 observations and has a mean of 12 and a standard deviation of 3. The standard error of the sampling distribution of the sample mean difference $\bar{X}_1 - \bar{X}_2$ is
(A) 0.1853 (B) 0.7602 (C) 0.7331 (D) 0.8719

6. If there are two unbiased estimators of a population parameter, the one whose variance is smaller is said to be:
(A) a biased estimator (B) relatively efficient (C) consistent (D) relatively unbiased

7. The power of a test is the probability that it will lead us to :
(A) reject the null hypothesis when it is true (B) reject the null hypothesis when it is false (C) fail to reject the null hypothesis when it is true (D) fail to reject the null hypothesis when it is false

8. For statistical inference about the mean of a single population when the population standard deviation is unknown, the degrees for freedom for the t distribution equal $n-1$ because we lose one degree of freedom by using the:
(A) sample mean as an estimate of the population mean (B) sample standard deviation as an estimate of the population standard deviation (C) sample proportion as an estimate of the population proportion (D) sample size as an estimate of the population size

9. Two independent samples of sizes 25 and 35 are randomly selected from two normal populations with equal variances. In order to test the difference between the population means, the test statistic is:
(A) a standard normal random variable (B) approximately standard normal random variable (C) Student t distributed with 58 degrees of freedom (D) Student t distributed with 33 degrees of freedom

10. The value of the test statistic in a completely randomized design for ANOVA is $F = 6.29$. The number of degrees of freedom for the numerator and denominator are 5 and 10, respectively. The most accurate statements to be made about the $p-$value is that it is:
(A) greater than 0.05 (B) between 0.025 and 0.05 (C) between 0.01 and 0,025 (D) smaller than 0.01

Part II: Problems (75 points)

1. (10 points) Let X represent the number of times a student visits a bookstore in a 1-month period. Assume that the probability distribution of X is as follows:

X	0	1	2	3
$P(X)$	0.05	0.25	0.50	0.20

   (a) Find the mean μ and the standard deviation σ of this distribution.

   (b) Find the mean and the standard deviation of $Y = 2X - 1$.

   (c) What is the probability that the student visits the bookstore at least once in a month?

   (d) What is the probability that the student visits the bookstore at most twice in a month?

2. (10 points) An analysis of the stock market produces the following information about the returns of two stocks.

	Stock 1	Stock 2
Expected Returns	15%	18%
Standard Deviations	20	32

   Assume that the returns are positively correlated with $\rho_{12} = 0.80$.

   (a) Find the mean and standard deviation of the return on a portfolio consisting of an equal investment in each of the two stocks.

   (b) Suppose that you wish to invest \$1 million. Discuss whether you should invest your money in stock 1, stock 2, or a portfolio composed of an equal amount of investments on both stocks.

3. (10 points) The time it takes a student to finish a chemistry test is uniformly distributed between 50 and 70 minutes.

   (a) What is the probability density function for this uniform distribution?

   (b) Find the probability that a student will take between 40 and 60 minutes to finish the test.

   (c) Find the probability that a student will take no less than 55 minutes to finish the test.

   (d) What is the expected amount of time it takes a student to finish the test?

   (e) What is the standard deviation for the amount of time it takes a student to finish the test?

4. (17 points) An ardent fan of television game shows has observed that, in general, the more educated the contestant, the less money he or she wins. To test her belief she gathers data about the last eight winners of her favorite game show. She records their winnings in dollars and the number of years of education. The results are as follows.

Contestant	Years of Education	Winnings
1	11	750
2	15	400
3	12	600
4	16	350
5	11	800
6	16	300
7	13	650
8	14	400

(a) Determine the least squares regression line.

(b) Interpret the value of the slope of the regression line.

(c) Determine the standard error of estimate and describe what this statistic tells you about the regression line.

(d) Determine the coefficient of determination and discuss what its value tells you about the two variables.

5. (18 points) A first- order model was used in regression analysis involving 25 observations to study the relationship between a dependent variable y and three independent variables x_1, x_2, and x_3. The analysis showed that the mean squares for regression is 160 and the sum of squares for error is 1050. In addition, the following is a partial computer printout.

Predictor	Coef	Std. Dev.
Constant	25	4
x_1	18	6
x_2	-12	4.8
x_3	6	5

(a) Develop the ANOVA table..

(b) Explain how to determine whether the model is useful in predicting the value of y at the 5% significance level.

(c) Explain how to determine whether x_1 is linearly related to y at the 5% significance level.

6. (10 points) An economist is analyzing the incomes of professionals (physicians, dentists, and lawyers). He realizes that an important factor is the number of years of experience. However, he wants to know if there are differences among the three professional groups. He takes a random sample of 125 professionals and estimates the multiple regression model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \epsilon$$

where

$$\begin{aligned} y &= \text{annual income (in \$1,000)} \\ x_1 &= \text{years of experience} \\ x_2 &= \text{1 if physician; } = 0 \text{ if not.} \\ x_3 &= \text{1 if dentist; } = 0 \text{ if not.} \end{aligned}$$

The computer output is shown below.

$$\underset{(18.56)}{y = 71.65} + \underset{(0.81)}{2.07}\ x_1 + \underset{(3.16)}{10.16}\ x_2 - \underset{(2.85)}{7.44}\ x_3 \qquad R^2 = 0.309$$

Note: standard errors in parentheses.

(a) Can you conclude that physicians earn more on average than lawyers? Why? Please explain.

(b) Can you conclude that physicians earn more on average than dentist? Why? Please explain.