招 生 學 年 度	100	招 生 類 別	碩士班
系 所 班 別	應用數學系碩士班、應用數學系 統計碩士班		
科　　　　目	專業語文測驗		
注 意 事 項	滿分為 60 分；中英文綜合命題		

Translate problem 1-3 into Chinese and problem 4 into English. (15 pts. Each)

1. (15points)In our study of metric spaces, we defined a number of key ideas like contact point, limit points, closure of a set, etc. In each case, the definition rests on the notion of a neighbourhood, or what amounts to the same thing, the notion of an open set. These notions were in turn defined by using the metric in the given space.

2. (15points)The problem of taking limits behind the integral sign, or equivalently of integrating a convergent series term by term, is often encountered in analysis. In the classical theory of integration, it is proved that a sufficient condition for taking such a limit is that the series (or sequence) in question be uniformly convergent. We now examine the corresponding theorems for Lebesgue integrals, which constitute a rather far-reaching generalization of their classical counterparts.

3. (15points)The strong law of large numbers is the foundation of a mathematical theory of probability based on the concept of frequency. It makes better sense than the weak one and is indispensable for certain theoretical investigations. But the empiricist may regard it as an idealistic fiction. It is amusing to quote two eminent authors on the subject. Feller: ＂ The weak law of large numbers is of very limited interest and should be replaced by the more precise and more useful strong law of large numbers＂ .Van der Waerden: "The strong law of large numbers scarcely plays a role in mathematical statistics＂ .

4. (15points)實數系的連續性是分析學的基礎，對極限論，微積分乃至整個分析學具有無比的重要性。 從幾何角度理解，它代表實數全體佈滿整個數軸而沒有空隙，但從分析角度看，則有多種等價的表述方式。譬如，確界存在定理就是實數系連續性的表述之一。