1. (1) Vector $\vec{A}=(1,-2,2)=1\hat{i}-2\hat{j}+2\hat{k}$, please try to find the unit vector (單位向量) of $\vec{A}$. (5%)

(2) Two vectors $\vec{B}=(2,3,-4)$ and $\vec{C}=(1,-1,1)$. Known the θ=$\cos^{-1}(x)$ is the angle between this two vectors, please try to find the x. [**Hint:** Vector dot product] (5%)

2. Known a matrix $\mathbf{A}=\begin{bmatrix}\cos x & -\sin x\\ \sin x & \cos x\end{bmatrix}$, please try to find $\mathbf{A}^{-1}$, $(\mathbf{A}^{-1})^2$ and then induce to find $(\mathbf{A}^{-1})^n$. [**Ps.** $\mathbf{A}^{-1}$ is defined as Inverse Matrix of **A**, and $\mathbf{AA}^{-1}=\mathbf{I}$] [**Hint:** sin(a+b)=sina×cosb+sinb×cosa ; cos(a+b)=cosa×cosb-sina×sinb] (10%)

3. Please try to find the solution of each following ODE respectively.

(1) $y'=\dfrac{dy}{dx}=-\dfrac{2x\sin(3y)}{3x^2\cos(3y)}$ (10%)

(2) $y'+y=y^3$ (10%)

(3) $y''-3y'+2y=\sin(e^{-x})$ (10%)

(4) $(x-2)^2y''+3(x-2)y'+y=x$ (10%)

(5) Simultaneous ODE (10%)

$$x''(t)-4x(t)+y(t)-2y'(t)=t$$
$$2x'(t)+x(t)+y''(t)=0$$

[**Hint:** The method of differential operator (微分算子法或逆運算法)]

4. Known the $y_1=e^{2x}$ is one homogeneous solution of ODE $y''-4y'+4y=0$, please try to prove the $y_2=xe^{2x}$ is also the solution of this ODE by using the Reduction of order method (降階法) [**Hint:** Let $y_2=u\times y_1$ and $y_1$is a known solution] (10%)

5. Please try to prove following equation (eq.1) by using the Fourier series expansion method for a periodic function $f(x)=x^2$ where $f(x+2\pi)=f(x)$ and $-\pi\le x\le\pi$. [**Hint:** (i) $f(\pi)=\pi^2$; (ii) $\cos(n\pi)=(-1)^n$]

$$\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\ldots \quad \text{(eq.1)} \qquad (10\%)$$

6. Please try to find the solution (u(x,t)) of following PDE (wave equation) with B.C. and I.C.

$$\frac{\partial^2 u}{\partial t^2}=\alpha^2\frac{\partial^2 u}{\partial x^2} \text{ and } \lim_{x\to\infty}u(x,t)=0\ (t>0)$$

I.C. u(x,0)=0 and $u_t(x,0)=0$

B.C. $u(0,t)=f(t)=\begin{cases}\sin(t) & 0\le t\le 2\pi\\ 0 & \text{other}\end{cases}$

[**Ps.** $u_t\equiv\dfrac{\partial u}{\partial t}$] [**Hint:** (i) Laplace Transform; (ii) $\int_0^{2\pi}e^{-ax}\sin(x)dx=\dfrac{1-e^{-2\pi a}}{1+a^2}$] (10%)