

科目：微積分 適用：財金所

編號：354

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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一、填空题(共 60 分，每空格 5 分，不需列出計算過程)

1. Evaluate $\lim_{n \rightarrow \infty} \sqrt{n^2 + n} - \frac{n}{\sqrt{n^2 + n}}$ = _____.

2. Evaluate $\lim_{n \rightarrow \infty} \sqrt[n]{n} =$ _____.

3. Let $f(x, y) = e^{\frac{y}{x}} - 2xy^2 - 3$

(a) Find $f_x =$ _____.

(a) Find $\frac{dy}{dx} =$ _____ if $f(x, y) = 0$.

4. Find the maximum and minimum values of the function

$$f(x, y) = e^{xy} \text{ subject to the constraint } x^2 + y^2 = 8.$$

(b) maximum value of the function = _____.

(c) minimum value of the function = _____.

5. Evaluate $\int x \ln \sqrt{x} dx =$ _____.

6. Find the general solution of $\frac{dy}{dx} = e^{2x+3}$. That is, $y =$ _____.

7. If $\int_a^b f(x) dx = -7$ and $\int_a^b g(x) dx = 3$, find

$$\int_b^a [f(x) - g(x)] dx =$$
 _____.

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8. Find the minimum value = _____ of the function

$$f(x, y) = x^2 + y^2 - xy$$

9. When the fourth Taylor polynomial for $f(x) = \frac{x^2}{1+x^2}$ at $x=0$ is usedto approximate the integral $\int_0^1 \frac{x^2}{1+x^2} dx = \underline{\hspace{2cm}}$.10. Evaluate the integral: $\iint_R (x^2 + 4y) dA = \underline{\hspace{2cm}}$ where R is theregion bounded by the graphs of $y = 2x$ and $y = x^2$.

二、計算題(共 40 分，沒有列出計算過程者不予計分)

1. (20%) Suppose $a_n > 0$ for all n , $S_n = a_1 + a_2 + \cdots + a_n$, and
$$\sum a_n \text{ diverges. Prove that } \frac{a_n}{S_{n-1}} - \frac{1}{S_n} \text{ and deduce that}$$

$$\sum \frac{a_n}{S_n^2} \text{ converges.}$$
2. (10%) The total resistance R of three resistors with resistance R_1 , R_2 ,and R_3 , connected in parallel, is given by the relationship

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

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If R_1 , R_2 , and R_3 are measured at 100, 200, and 300 ohms, respectively, with a maximum error of 1% in each measurement, find the approximate maximum error in the calculated value of R .

3. (10%) A manufacturer is planning to sell a new product at the price of \$150 per unit and estimates that if x thousand dollars is spent on development and y thousand dollars is spent on promotion, approximately $\frac{320y}{y+2} + \frac{160x}{x+4}$ units of the product will be sold. The cost of manufacturing the product is \$50 per unit. If the manufacturer has a total of \$8,000 to spend on development and promotion, how should this money be allocated to generate the largest possible profit?