

科目：微積分 適用：財金系

編號：354

考生注意：

1. 依序作答，只要標明題號，不必抄題。

2. 答案必須寫在答案卷上，否則不予計分。

3. 限用藍、黑色筆作答；試題須隨卷繳回。

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第 1 頁

## 一、填空題(共 66 分，每空格 6 分，不需列出計算過程)

1. Evaluate  $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^n = \underline{\hspace{2cm}}$

2. Evaluate  $\lim_{x \rightarrow \infty} \frac{\ln x}{x} = \underline{\hspace{2cm}}$

3. Evaluate

$$\int_0^1 xe^x dx = \underline{\hspace{2cm}}$$

4. Let  $f(x, y) = y^x$   $\underline{\hspace{2cm}}$

(a) Find  $\frac{\partial^2 f}{\partial x^2} = \underline{\hspace{2cm}}$

(b) Find  $\frac{\partial^2 f}{\partial y^2} = \underline{\hspace{2cm}}$

5. Find the derivatives of  $f(x) = x^x$  ( $x > 0$ )

$$f'(x) = \underline{\hspace{2cm}}$$

6. Evaluate

$$\int_0^1 \int_0^{1-x} \sqrt{x + y} (\underline{\hspace{2cm}})^2 dy dx = \underline{\hspace{2cm}}$$

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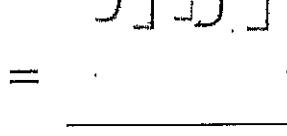
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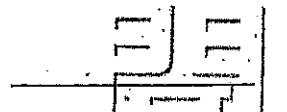
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7.  $z(x, y) = xy, x(u, v) = 2u - v, y(u, v) = \ln uv$

(a) Evaluate  $\frac{\partial z}{\partial u} =$  

(b) Evaluate  $\frac{\partial z}{\partial v} =$  

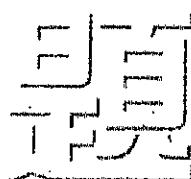
8. Find the power series representation about  $x = 0$  for the function

$f(x) = e^x =$  

9. Find the value(s) of  $a$  for which the series

$$\sum_{n=1}^{\infty} \left( \frac{a}{n+1} - \frac{1}{n+2} \right)$$

converges. Ans:  $a =$  



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## 二、計算題(共 34 分，沒有列出計算過程者不計分)

1. (10%) Evaluate the integral:  $\iint_R \sqrt{x} - y \, dA$  where  $R$  is the region

bounded by the graphs of  $y = x + 1$ ,  $y = x - 1$ ,  $y = \frac{1}{2}x$ , and

$$y = \frac{1}{2}x + 1$$

2. (a) (4%) Show that Simpson's rule is

$$\int_a^b f(x) \, dx \approx \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots + 4f(x_{n-1}) + f(x_n)]$$

where  $\Delta x = \frac{b-a}{n}$  and  $n$  is even.

- (b) (10%) Find approximations of

$$\int_0^1 \frac{1}{1+x} \, dx$$

by using the Simpson's rule with  $n=4$ .

3. (10%) By cutting away identical squares from each corner of a rectangular piece of cardboard and folding up the resulting flaps, the cardboard may be turned into an open box. If the cardboard is 16 inches long and 10 inches wide, find the dimensions of the box that will yield the maximum volume.