注意事項：

一、 請依照題號順序作答，且必須列出計算過程，否則不予計分。

二、 本試題總共六大題，各題配分標示於題號之後。

1. (15%) CCU Products, Inc., manufactures three products on two machines. Each product has to be routed through these two machines. In a typical week, 40 hours are available on each machine. The profit contribution and production time in hours per unit are as follows:

Category	Product 1	Product 2	Product 3
Profit/unit	$30	$50	$20
Machine M1 time/unit	0.5	2.0	0.75
Machine M2 time/unit	1.0	1.0	0.5

Two operators are required for machine M1; thus, 2 hours of labor must be scheduled for each hour of machine M1 time. Only one operator is required for machine M2. A maximum of 100 labor hours is available for assignment to the machine during the coming week. Other production requirements are that Product 1 cannot account for more than 50% of the units produced and that product 3 must account for at least 20% of the units produced. Develop a linear programming (LP) model (including the objective function and the constraints) for this problem. The objective is to maximize profit. **Note that you must define each decision variable clearly.**

2. (20%) The computer output (from Lingo) for problem 1 is as follows:
Suppose Xi denotes the production quantity of product i (i = 1, 2, 3).
Constraint 1 is the limitation for M1 machine hour.
Constraint 2 is the limitation for M2 machine hour.
Constraint 3 is the limitation for labor hour.
Constraint 4 is the limitation for product 1 requirement.
Constraint 5 is the limitation for product 3 requirement.

Objective value: 1250.000

Variable	Value	Reduced Cost
X1	25.00000	0.000000
X2	0.000000	7.500000
X3	25.00000	0.000000

constraint	Slack or Surplus	Dual Price
1	8.750000	0.000000
2	2.500000	0.000000
3	0.000000	12.50000
4	0.000000	10.00000
5	15.00000	0.000000

Ranges in which the basis is unchanged:

Objective Coefficient Ranges

Variable	Current Coefficient	Allowable Increase	Allowable Decrease
X1	30.00000	INFINITY	10.00000
X2	50.00000	7.500000	INFINITY
X3	20.00000	10.00000	4.285714

Righthand Side Ranges

Constraint	Current RHS	Allowable Increase	Allowable Decrease
1	40.00000	INFINITY	8.750000
2	40.00000	INFINITY	2.500000
3	100.0000	6.666667	100.0000
4	0.0	5.000000	25.00000
5	0.0	15.00000	INFINITY

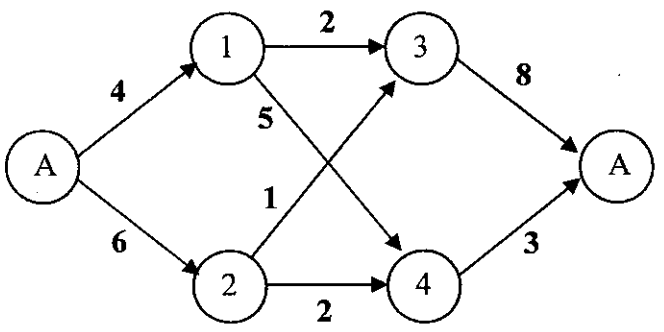
a. Based on the computer output, the reduced cost of X2 is 7.5. What is the meaning of this number (i.e., 7.5)?
b. What is/are the *binding* constraint(s) in this LP model?
c. Suppose the labor capacity can be increased to 105 hours. What will be the profit?
d. Suppose the unit profit of product 1, 2, 3 is decreased by 5, 8, and 4, respectively. What is the new optimal profit?

3. (15%) The ABC Bank is working to develop an efficient work schedule for full-time and part-time tellers. The schedule must provide for efficient operation of the bank including adequate customer service, employee breaks, and so on. On Fridays the bank is open from 9:00 a.m. to 7:00 p.m. The number of tellers necessary to provide adequate customer service during each hour of operation is summarized below.

Time	9-10	10-11	11-12	12-1	1-2	2-3	3-4	4-5	5-6	6-7
No. of Tellers	6	5	8	10	8	7	5	7	7	6

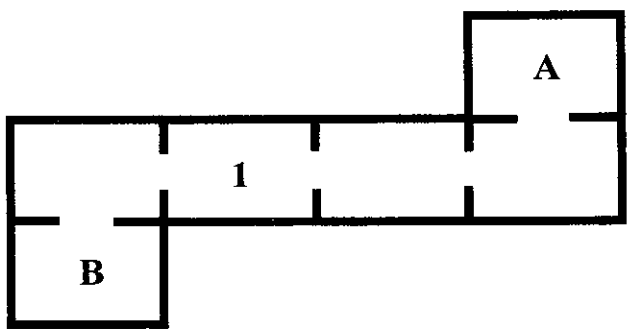
Each full-time employee starts on the hour and works a 4-hour shift followed by 1 hour for lunch and then a 3-hour shift. Part-time employees work one 4-hour shift beginning on the hour. Considering salary and fringe benefits, full-time employees cost the bank $15 per hour ($105 per day), and part-time employees cost the bank $8 per hour ($32 per day). Formulate an integer programming model that can be used to develop a schedule that will satisfy customer service needs at a minimum employee cost. **Note that you must define each decision variable clearly.**

4. (20%) Use dynamic programming technique to find the shortest path from node A to node B for the following network: **(Be sure to define the recursive function and show your work)**



5. (10%) Buses arrive at a bus stop according to a Poisson distribution with rate $\mu = 3$ per hour. It is known that a bus arrived during the past 20-minute period from 8:20 to 8:40 AM. What is the probability that a man waited from 8:25 to 8:30 AM will catch the bus?

6. (20%) A rat is put into compartment 1 of the following maze:



The rat moves to the neighbor compartments at random.

a. What is the probability that the rat reaches compartment A before B?
b. What is the expected movements that the rat reaches compartment A?