1. (15 %) Two discrete-time signals $x[n]$ and $h[n]$ are shown in Figure 1

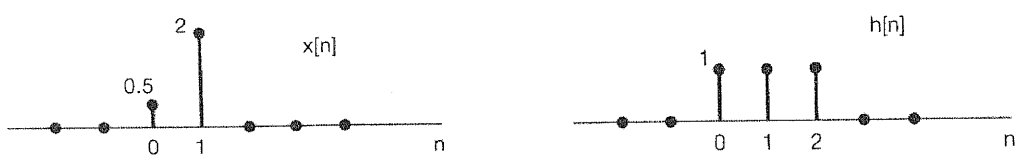

Figure 1

(A) (5 %) Let $X(e^{j\omega})$ be the Fourier transforms of $x[n]$. Determine $X(e^{j0})$.

(B) (10 %) Plot the convolution sum of $x[n]$ and $h[n]$.

2. (20 %) Let $x(t)$ be a real-values signal with fundamental period T and Fourier series coefficients a_k.

(A) (5 %) Show that $a_k = a_{-k}^*$ and a_0 must be real.

(B) (5 %) Show that if $x(t)$ is even, then its Fourier series coefficients must be real and even.

(C) (5 %) Show that if $x(t)$ is odd, then its Fourier series coefficients are imaginary and odd and $a_0 = 0$.

(D) (5 %) Show that the Fourier coefficients of the even part of $x(t)$ are equal to the real part of a_k.

3. (15 %) Compute the convolution of each of the following pairs of signals $x(t)$ and $h(t)$.

(A) (5 %) $x(t) = te^{-2t}u(t),\ h(t) = e^{-4t}u(t)$

(B) (5 %) $x(t) = te^{-2t}u(t),\ h(t) = te^{-4t}u(t)$

(C) (5 %) $x(t) = e^{-t}u(t),\ h(t) = e^{t}u(-t)$

4. (15 %) Consider the system shown in Figure 2 with input $x[n]$ and output $y[n]$, where $H_{lp}(e^{j\omega})$ is an ideal lowpass filter with cutoff frequency $\pi / 6$ and unit gain in the passband.

(A) (7%) Determine the overall frequency response of the system and plot it.

(B) (8%) Determine the impulse response of the overall system.

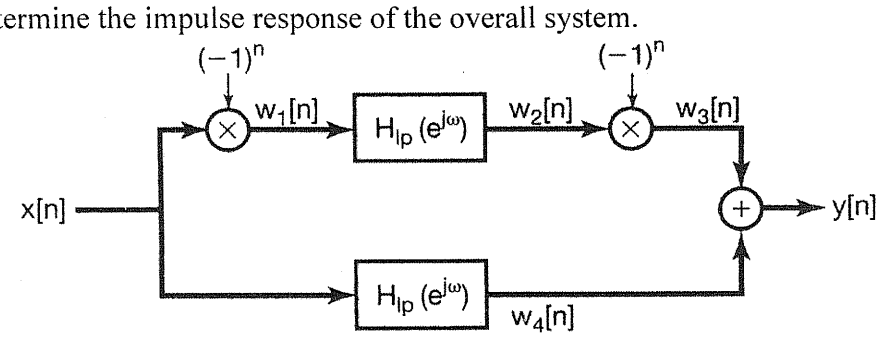

Figure 2

5. (15 %) Consider a causal discrete-time LTI system whose input $x[n]$ and output $y[n]$ are related by the following difference equation:

$$y[n]=\frac{1}{2}y[n-1]+\frac{1}{4}x[n]$$

Find the **Fourier series representation** of the output $y[n]$ for each of the following inputs:

(A) (7 %) $x[n]=\sin\left(\frac{3\pi n}{4}\right)$

(B) (8 %) $x[n]=\cos\left(\frac{\pi n}{4}\right)+2\cos\left(\frac{\pi n}{2}\right)$

6. (20 %) Figure 3 depicts a block diagram implementation of a causal LTI system.
(A) (6 %) Find a difference equation relating $x[n]$ and $y[n]$ for this system.
(B) (7 %) What is the frequency response of the system?
(C) (7 %) Determine the system's impulse response.

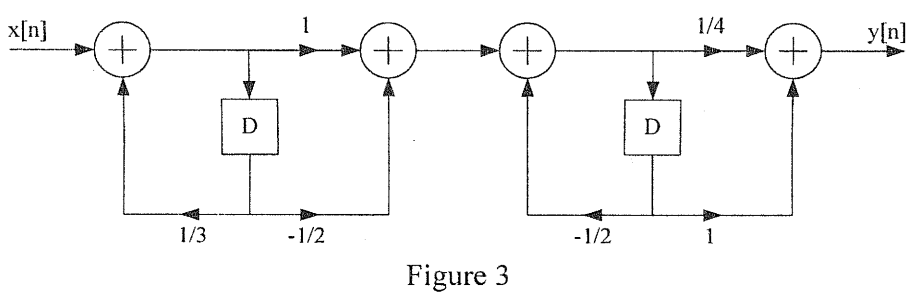

Figure 3