1. (50分) 選擇題(每題2.5分，20題共50分。答錯1題倒扣0.5分，不答不給分。)

Useful constants:

Speed of light = 3.0×10^{8} m/s　　Planck's constant = 6.6×10^{-34} J·s

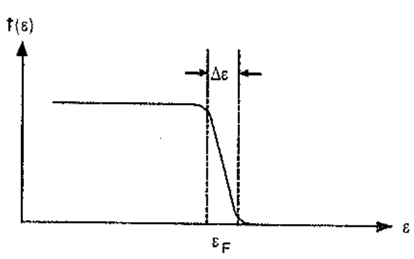
Boltzmann constant (k) = 1.38×10^{-23} J/K　　1 eV = 1.6×10^{-19} J

(1) Which of the following experiments independently determines the charge of an electron?
(A) Millikan oil-drop experiment (B) Thomson experiment (C) Franck-Hertz experiment
(D) Cyclotron resonance (E) Compton effect.

(2) Which of the following would be a true statement about the Frank-Hertz experiment?
(A) the value of Planck's constant was first measured
(B) the charge to mass ratio of the electron was measured
(C) it was proved that atomic energy states are quantized
(D) it was proved that the electron has spin
(E) the quantization of photon energy was discovered

(3) The longest wavelength x-ray that can undergo Bragg diffraction in a crystal for a given family of planes of spacing d is (A) $d/4$ (B) $d/2$ (C) d (D) $2d$ (E) $4d$

(4) The Fermi distribution function f(ε) for electrons in a metal is shown below. Which of the following statements is true for this metal?
(A) The Fermi energy ε_F is of the order of kT.
(B) The spread in energies Δε is independent of the temperature.
(C) The higher the density of elections, the lower the Fermi level.
(D) The distribution results from the assumption that any number of elections can occupy a given quantum state.
(E) Only those elections which ~ kT of the Fermi level can be excited thermally.



(5) Which of the following is not a true statement about nucleons?
(A) protons and neutrons are fermions
(B) even Z even N nuclei have zero total angular momentum
(C) the total angular momentum is integral for nuclei with even A
(D) protons and neutrons have integer spin
(E) the total angular momentum is half-integral for nuclei with odd A

(6) The observed specific heat of the electrons in a metal is much smaller than classical (i.e., non-quantum) statistical mechanics would indicate. The reason for this is directly related to
(A) special relativity (B) the Pauli exclusion principle (C) the indeterminacy principle
(D) Hund's rule (E) the principle of least action

(7) A beam of particles is incident on a thin target of thickness t. If the cross section per nucleus for scattering of the

particles by the nuclei of the target is σ and the number of nuclei per unit volume is n, the fraction of particles being scattered by the target is

(A) σ / nt　(B) σt　(C) nσ　(D) nt / σ　(E) ntσ

(8) Which of the following is <u>not</u> characteristic of a superconductor?
(A) The resistivity vanishes below the transition temperature.
(B) A sufficiently large magnetic field can destroy the superconducting state.
(C) A magnetic field is excluded from the superconductor.
(D) A gap exists in the allowable energy levels of the material.
(E) The superconductor is paramagnetic.

(9) The total energy of a blackbody radiation source is collected for one minute and used to heat water. The temperature of the water increases from 25.0°C to 26.0°C. If the absolute temperature of the blackbody source were to be doubled and the experiment repeated, which of the following statements would be mostly correct?
(A) The temperature of the water would increase from 25°C to a final temperature of 26°C.
(B) The temperature of the water would increase from 25°C to a final temperature of 28°C.
(C) The temperature of the water would increase from 25°C to a final temperature of 35°C.
(D) The temperature of the water would increase form 25°C to a final temperature of 41°C.
(E) The water would boil within the one-minute period.

(10) A beam of neutral hydrogen atoms in their ground state moving into the plane of this page and passes through a region of a strong inhomogeneous magnetic field that is directed upward in the plane of the page. After the beam passes through this field, a detector would find that it has been
(A) deflected upward　(B) deflected to the right　(C) split vertically into two beams
(D) split horizontally into three beams　(E) undeviated

(11) If singly ionized helium atom in an $n = 4$ state emits a photon of wavelength 470 nanometers, which of the following gives the approximate final energy level E_f of the atom and the n value n_f of this final state?

	E_f (eV)	n_f
(A)	-6.0	3
(B)	-6.0	2
(C)	-14	2
(D)	-14	1
(E)	-52	1

(12) A $3p$ electron is found in the $^2P_{3/2}$ energy level of a hydrogen atom. Which of the following is true about the electron in this state?
(A) It is allowed to make an electric dipole transition to the $^2S_{1/2}$ level.
(B) It is allowed to make an electric dipole transition to the $^2P_{1/2}$ level.
(C) It has quantum numbers $l = 3, j = 3/2, s = 1/2$.
(D) It as quantum number $n = 3, l = 1, s = 3/2$.
(E) It has exactly the same energy level as in the $^2D_{3/2}$ level.

(13) Light of wavelength 500 nanometers is incident on sodium, which has a work function of 2.28 eV. What is the maximum kinetic energy of the ejected photoelectrons?

(A) 0.2 eV (B) 0.4 eV (C) 0.6 eV (D) 1.3 eV (E) 2.0 eV

(14) Two electrons, e.g., those in an excited state of He, interact by a Coulomb potential. If their spins are parallel, the spatial part of the total wave function must be antisymmetric with respect to exchange. This triplet state is lower in energy than the corresponding singlet state (antiparallel spins) because in the triplet state the
(A) magnetic dipole-dipole interaction is weaker (B) magnetic dipole-dipole interaction is stronger
(C) electrons are on the average closer together (D) electrons are on the average farther apart
(E) spin-orbit couplings are weaker

(15) In an *n*-type semiconductor, which of the following statements is true regarding the impurity atoms?
(A) Electrons from the valence band fill those atoms' empty energy levels located just above the valence band.
(B) Electrons from the valence band fill those atoms' empty energy levels located just below the valence band.
(C) Electrons from the valance band fill those atoms' empty energy levels located just above the conduction band.
(D) Electrons from the valance band fill those atoms' empty energy levels located just below the conduction band.
(E) They donate electrons to the conductor band from filled donor levels just below the conduction band.

(16) For an ideal diatomic gas at equilibrium, the ratio of the molar heat capacity at constant volume at very high temperature to that at very low temperature is equal to
(A) 1 (B) 5/3 (C) 2 (D) 7/3 (E) 4

(17) A system consists of N weakly interacting subsystems, each with two internal quantum states with energies 0 and ε. The internal energy for this system at absolute temperature T is equal to
(A) $N\varepsilon$ (B) $(3/2)NkT$ (C) $N\varepsilon\exp(-\varepsilon/kT)$
(D) $N\varepsilon/[1+\exp(\varepsilon/kT)]$ (E) $N\varepsilon/[1-\exp(-\varepsilon/kT)]$

(18) The wave function for identical fermions is anti-symmetric under particle interchange. Which of the following is a consequence of this property?
(A) Pauli exclusion principle (B) Bohr correspondence principle
(C) Heisenberg uncertainty principle (D) Bose-Einstein condensation
(E) Fermi's golden rule

(19) The electronic energy levels of atoms of a certain gas are given by $E_n = E_1 \cdot n^2$, where n = 1, 2, 3, ... Assume that transitions are allowed between all levels. If one wanted to construct a laser from this gas by pumping the n = 1 → n = 3 transition, which energy level or levels would have to be metastable?
(A) n = 1 only (B) n = 2 only (C) n = 1 and n = 3 only
(D) n = 1, n = 2, and n = 3 (E) None

(20) According to Bose-Einstein statistics, there exists a Bose condensate for collections of bosons. What does this mean?
(A) as $T \rightarrow \infty$ all particles reside in excited states (B) for $T < T_C$ all particles reside in the ground state
(C) bosons are not physically meaningful particles (D) bosons are like fermions
(E) for $T < T_C$ bosons dissolve into quarks and gluons

2. A particle of mass M, initially at rest, decays into two particles, each of mass m.
   (a) (10 points) What is the speed of each outgoing particle as it flies off?
   (b) (10 points) Find the magnitudes of the outgoing relativistic momenta.

3. The time-dependent Schrödinger equation for a freely rigid rotator is

$$i\hbar\frac{\partial}{\partial t}\Psi(\phi,t)=-\frac{\hbar^2}{2I}\frac{\partial^2}{\partial\phi^2}\Psi(\phi,t),$$

   where I is the rotational inertia, and $\Psi(\phi,t)$ is the wave function written in terms of the angular coordinate ϕ and the time t.
   (a) (5 points) By applying the technique of separation of variables $\Psi(\phi,t)=\Phi(\phi)T(t)$ and introducing a separation constant E to obtain (i) the time-independent Schrödinger equation for $\Phi(\phi)$ and (ii) the equation for the time dependence of the function $T(t)$.
   (b) (5 points) Solve the equation for the time dependence of the function $T(t)$ obtained in (ii) of part (a) and then show that the separation constant E is the total energy.
   (c) (5 points) Show that $\Phi(\phi)=e^{im\phi}$ is a particular solution to the time-independent Schrödinger for the rigid rotator in (i) of part (a). Find the value m in terms of I and E.
   (d) (5 points) Apply the condition of single valuedness to the particular solution of part (c) to find the allowed values of the total energy E for the quantum rigid rotator.

4. (10 points) Enumerate the possible values of the **total** angular momentum quantum number j and m_j, for the coupled states in which the orbital angular momentum $l=2$ and spin angular momentum $s=1/2$.