請依題號順序作答，並且將答案填寫於答案卷上．

1. Municipal bond service in Country Avatar has three rating categories (A, B, and C). Suppose that in the past year, of the municipal bonds issued, 60% were rated A, 25% were rated B, and 15% were rated C. Of the municipal bonds rated A, 55% were issued by cities, 40% by suburbs, and 5% by rural area. Of the municipal bonds rated B, 70% were issued by cities, 15% by suburbs, and 15% by rural areas. Of the municipal bonds rated C, 85% issued by cities, 10% by suburbs, and 5% by rural areas.
   a) What proportions of municipal bonds are issued by cities, suburbs, and rural areas, respectively? (12 points)
   b) If a new municipal bond is to be issued by a suburb, what is the probability that it will be rated A? (4 points)

2. Assume random variable, X is distributed as a binomial distribution with *n* trials and probability of success, *p*.
   a) Write down the probability distribution function. (2 points)
   b) Interpret "expected value." (2 points)
   c) Find the expected value of X, E(X) = *np*. (10 points)
   (Hint: provided $x \neq 0$, we can expand $x!$ as $x(x-1)!$. $\sum_{k=0}^{n}\binom{n}{k}p^k = (1+p)^n$)

3. A local pizza restaurant and a local branch of a national chain are located across the street from a college campus. The local pizza restaurant advertises that they deliver to the dormitories faster than the national chain. The delivery time of these two restaurants are summarized as follows.
   a) Explain "Type I Error." (3 points)
   b) Assuming the probability of Type I Error is 10%, can you verify the claim of local pizza restaurant? (17 points)

	Local	Chain
Mean (minute)	16.7	18.88
Standard Dev.	3.0955	2.8662
Observations	12	12

4. Suppose that X is a discrete random variable taking values in the non-negative integers {0, 1, 2, ...}. Let $G(s) = E(s^X)$ denote the probability generating

function of X. Find the mean of X in terms of the probability generating function $G(s)$. (5 points)

5. Suppose that $\mathbb{y}$ is a constant vector in $\mathbb{R}^n$, $\mathbb{X}$ is a given $n \times k$ matrix, and $\mathbb{b}$ is an unknown vector in $\mathbb{R}^k$. Find the value of $\mathbb{b}$ that minimizes the norm of the vector $\mathbb{y} - \mathbb{X}\mathbb{b}$. (10 points)

6. Suppose that X and Y follow a bivariate normal distribution, the correlation coefficient between X and Y is ρ, the marginal distribution of X is normal with mean 0 and standard deviation σ_X, and the marginal distribution of Y is normal with mean 0 and standard deviation σ_Y. Find the conditional variance of Y given $X = x$, where x is a constant. (10 points)

7. Let $Y_1, Y_2, \ldots, Y_n$ be a random sample of observations from a uniform distribution with probability density function $f(y_i|\theta) = 1/\theta$, for $0 \le y_i \le \theta$ and $i = 1, 2, \ldots, n$. Find the maximum likelihood estimator of the parameter θ. (10 points)

8. Suppose that $X_1, X_2, \ldots, X_n$ form a random sample from a distribution for which the mean μ and the variance σ^2 are finite. Let $\bar{X}_n$ denote the sample mean. Use the Chebyshev inequality to prove that $\bar{X}_n$ converges to μ in probability. (10 points)

9. Suppose X and Y are random variables defined on the same probability space. Also suppose $E(Y|X) = \alpha + \beta X$, where α and β are constants. Use the law of iterated expectations to prove that $\alpha = E(Y) - \beta E(X)$. (5 points)