國立政治大學九十七學年度研究所~~博~~碩士班入學考試命題紙

考試科目	統計學	所別	風管所精算組	考試時間	3月16日 星期四 第3節

1. Please explain the following items. (40%)

   (a) Efficiency, Completeness and Uniqueness, (15%)

   (b) The Likelihood Ratio Test, (10%)

   (c) The Bayes' Solution, (5%)

   (d) The Central Limit Theorem, (5%)

   (e) The Rao-Cramer Inequality. (5%)

2. Given $f(x;\theta)=1/\theta,\ 0<x<\theta$, zero elsewhere, with $\theta>0$, formally compute the reciprocal of $nE\left\{\left[\frac{\partial \ln f(X;\theta)}{\partial\theta}\right]^2\right\}$. Compare this with the variance of $(n+1)Y_n/n$, where Y_n is the largest item of a random sample of size n from this distribution. Comment on the findings. (10%)

3. If X_1, X_2 is a random sample of size 2 from a distribution having p.d.f. $f(x;\theta)=(1/\theta)e^{-x/\theta}$, $0<x<\infty$, $0<\theta<\infty$, zero elsewhere, find the joint p.d.f. of the sufficient statistic $Y_1=X_1+X_2$ for θ and $Y_2=X_2$. Show that Y_2 is an unbiased estimator of θ with variance θ^2. (10%)

4. Let X_1, X_2 be a random sample from the normal distribution N(0,1). Find the marginal p.d.f. of $Y_1=X_1/X_2$. (10%)

備考	試題隨卷繳交
命題委員：	（簽章）108年3月7日

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5. Let $Y_1 < Y_2 < Y_3$ be the order statistics of a random sample of size 3 from a distribution having the p.d.f. $f(x) = 2x$, $0 < x < 1$, zero elsewhere. Show that $Z_1 = Y_1/Y_2$, $Z_2 = Y_2/Y_3$, and $Z_3 = Y_3$ are mutually stochastically independent. (10%)

6. Let X have a p.d.f. of the form $f(x;\theta) = 1/\theta$, $0 < x < \theta$, zero elsewhere. Let $Y_1 < Y_2 < Y_3 < Y_4$ denote the order statistics of a random sample of size 4 from this distribution. Let the observed value of Y_4 be y_4. We reject $H_0 : \theta = 1$ and accept $H_1 : \theta \neq 1$ if either $y_4 \leq \frac{1}{2}$ or $y_4 \geq 1$. Find the power function of the test. (10%)

7. Let $\bar{X}_n$ denote the mean of a random sample of size n from a gamma distribution with parameters $\alpha = \mu > 0$ and $\beta = 1$. Find the limiting distribution of $\sqrt{n}(\bar{X}_n - \mu)/\sqrt{\bar{X}_n}$. (10%)

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