

考試科目	微積分	所別	財管所 4171	考試時間	3月16日 星期日	第三節
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1. (8 points each) Evaluate the following limits.

$$(i) \lim_{\theta \rightarrow 0} \frac{\sin \theta - \tan \theta}{\theta^3} \quad (ii) \lim_{n \rightarrow \infty} n e^{-n^2} \int_0^n e^{x^2} dx$$

2. (8 points each) Evaluate the following integrals.

$$(i) \int_{-3}^3 |x^3 - x| dx \quad (ii) \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \cos x \sin y dx dy \quad (iii) \int_0^e y dx, \text{ where } e^y + y = x + 1.$$

3. (15 points) Suppose that $\lim_{x \rightarrow 2} \frac{f(x)}{x-2} = 2$, $\lim_{x \rightarrow 3} \frac{f(x)}{x-3} = 3$ and $\lim_{x \rightarrow 4} \frac{f(x)}{x-4} = 4$. Find the polynomial $f(x)$ of least degree.

4. (i) (10 points) Suppose that $x > 0$. Find the maximum of $f(x) = \frac{\ln x}{x}$.

(ii) (10 points) Apply the result of (i) to prove that $e^\pi > \pi^e$.

5. (15 points) Suppose that $y(x)$ satisfies the differential equation $x^2 y' + (3x^2 + 2)y^2 = 0$ and the initial condition $y(1) = \frac{1}{3}$. Solve $y(x)$.

6. (10 points) The marginal cost function of producing x units of widgets is given by the function $MC(x) = \frac{1}{4}x + 50$. If the total cost of producing 20 units is 1500, then determine the total cost function $C(x)$.

備考	試題隨卷繳交
命題委員：	(簽章) 2008年3月7日

命題紙使用說明：1. 試題將用原件印製，敬請使用黑色墨水正楷書寫或打字（紅色不能製版請勿使用）。
 2. 書寫時請勿超出格外，以免印製不清。
 3. 試題由郵寄遞者請以掛號寄出，以免遺失而示慎重。