國立政治大學九十 七 學年度研究所(博/碩)士班入學考試命題紙

考試科目	數理統計	所別	統計 4141	考試時間	3月16日 星期日 第三節

1. Let $Y_1 < Y_2 < Y_3 < \cdots < Y_n$ be the order statistics of a random sample of size n from exponential distribution with p.d.f. $f(x) = e^{-x}$, $0 < x < \infty$, zero else where.

   (a) Show that $Z_1 = nY_1$, $Z_2 = (n-1)(Y_2 - Y_1)$, …, $Z_n = Y_n - Y_{n-1}$ are independent and each Z_i has the exponential distribution. (8%)

   (b) Demonstrate that all linear function of $Y_1, Y_2, \ldots, Y_n$, such that $\sum_{i=1} a_i Y_i$ can be expressed as linear function of independent random variables. (7%)

2. If we have a random sample $X_1, X_2, \ldots, X_n$ from a p.d.f.

   $f(x) = \frac{\theta^\kappa}{\Gamma(\kappa)} \exp(\kappa x - \theta e^x)$ for $-\infty < x < \infty$.

   (a) If $\kappa = 1$, find a complete and sufficient statistic , V , for θ. (3%)

   (b) If $\kappa = 1$. Let $W_i = e^{X_i}$ for $i = 1,2,\ldots,n$, $\overline{W} = \frac{\sum W_i}{n}$ and $U = \prod_{i=1} \frac{W_i}{\overline{W}}$.

   Show that U is independent of V. (5%)

   (c) Suppose that Q is a test for testing $H_0 : \kappa \le 1$ versus $H_1 : \kappa > 1$. What is an Unbiased test? (5%)

   (d) If Q is an unbiased test, show that $E(Q \mid V) = \alpha$ almost everywhere when $\kappa = 1$. (5%)

   (e) State how to construct a UMP unbiased test for $H_0 : \kappa \le 1$ versus $H_1 : \kappa > 1$. (5%)

3. Let $X_1, X_2, \ldots, X_n$ be a random sample from $EXP(\theta, \eta)$, and let $Z_i = (X_i - \eta)/\theta$. Find the BLUEs (Best Linear Unbiased Estimator) for θ and η. (12%)

備考	試題隨卷繳交

命題委員：　　(簽章) 97年 3月 8日

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國立政治大學 九十七學年度 碩士班暨碩士在職專班招生考試 命題紙

考試科目	數理統計學	所別	統計 414	考試時間	3 月 16 日 星期 日 第 三 節

4、Suppose the joint pdf of X and Y is $f(x,y)=\begin{cases} e^{-y}, & y>x>0 \\ 0, & otherwise \end{cases}$.

a) Find the marginal pdf of X. (3%)

b) Find $E(Y|x)$. (5%)

c) Find the correlation coefficient of X and Y. (8%)

5、Consider a random sample of size n form a population with pdf: $f(x)=(1-p)^{x-1}p$, $0<p<1$, $x=1,2,3,...$

a) Show that $1/\bar{X}_n$ converges in probability to P(X=1). (5%)

b) Show that $(\bar{X}_n-1)/\bar{X}_n^2$ converges in probability to P(X=2). (5%)

c) Find the asymptotic distribution of $(\bar{X}_n-1)/\bar{X}_n^2$. (10%)

6、Let X be the lifetime of a machine. Suppose that the pdf of X is $f(x)=\frac{x^{\kappa-1}}{\Gamma(\kappa)\beta^{\kappa}}e^{-x/\beta}$, $x>0$ where $\beta,\kappa>0$.

a) Find the mode of X. (6%)

b) Let $h(x)=a\exp(-cx)$ be the cost function of operating this machine to the lifetime $X=x$, where $a>0$. In order that the expected cost of operation of this machine exists, what is the range for the constant c? What is the expected cost? (8%)

備考	試題隨卷繳交
命題委員：	（簽章） 97 年 3 月 6 日

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