財務工程與金融創新組

國立政治大學九十七學年度研究所博碩士班入學考試命題紙 第1頁，共1頁

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考試科目	數理統計	所別	金融(財工)	考試時間	3月16日 星期日 第三節

1. Consider a random sample $X_1, X_2, \cdots, X_n$ from a discrete distribution with pdf $f(x;\theta)=\frac{(\theta+1)^x}{(\theta+2)^{x+1}}$ if $x=0,1,\ldots$ where $\theta>0$. Find a UMP test of $H_0:\theta=\theta_0$ against $H_a:\theta>\theta_0$. (10%)

2. Let $X \sim Poi(\lambda)$, and let $\theta=P(X\le 1)$.
   (a) Find an unbiased estimator of θ. (6%)
   (b) Find a sufficient statistic for θ. (5%)
   (c) Find a UMVUE of θ. (7%)

3. If $X_1, X_2, \cdots, X_n$ is a random sample from a distribution with p.d.f. $f(x;\theta)=\theta^2 x e^{-\theta x}$, $0<x<\infty$, zero elsewhere, where $0<\theta<\infty$;
   (a) Find a complete sufficient statistic for θ. (5%)
   (b) Show that $X_1/\sum_i X_i$ and $\sum_i X_i$ are independent. (8%)
   (c) What is the distribution of $X_1/\sum_i X_i$? (12%)

4. Consider a simple linear regression model, $y_i=\beta_0+\beta_1 x_i+\varepsilon_i$, where $E(\varepsilon_i)=\mu\neq 0$, $Var(\varepsilon_i)=\sigma^2$, β_0 and β_1 are unknown parameters.

   (a) If we use the traditional least squares estimator of β_0 and β_1 $(\hat{\beta}_0,\hat{\beta}_1)$, will the estimators still have the unbiasedness? (12%)

   (b) $Cov(\hat{\beta}_1,\bar{y})=?$ (6%)

5. Let $Y_i \sim LOGN(\mu_i,\sigma_i^2)$ $i=1,\ldots,n$ be independent. Find the distribution of :

   (a) $\prod_{i=1}^{n} Y_i$. (4%)

   (b) Y_1/Y_2. (5%)

   (c) Find $E\left[\prod_{i=1}^{n} Y_i\right]$. (5%)

6. Let X be a Negative Binomial random variable with parameter k=3 and p=0.3.
   (a) Compute the moment generating function of X. (8%)
   (b) What is the limiting distribution of X as $k\to\infty$, $p\to 1$ and $k(1-p)=\lambda$ (constant). (7%)