

考試科目	微積分	所別	金融學 4123 財稅工程與金融創 新組	考試時間	3月16日 星期日	第3節
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Instructions: Answer All Questions.

1. [25 Points]

(a) (10 Points) Let

$$f(x, y, z) = \frac{z \cdot \sin y}{x}$$

where $x = 3r^2 + 2s$, $y = 4r - 2s^3$, $z = 2r^2 - 3s^2$.

Find

$$\frac{\partial f}{\partial r}$$

and

$$\frac{\partial f}{\partial s}$$

(b) (15 Points) If

$$f(\lambda x, \lambda y) = \lambda^n f(x, y)$$

for some constant n , and all real values of λ .

Show that

$$nf(x, y) = x \left(\frac{\partial f}{\partial x} \right) + y \left(\frac{\partial f}{\partial y} \right)$$

2. [25 Points] Evaluate the following integrals:

(a) (10 Points)

$$\int \cos(x - \ln x) \left(\frac{x-1}{x} \right) dx$$

(b) (15 Points)

$$\int_0^3 \int_{x^2}^9 x^3 \exp(y^2) dy dx$$

3. [25 Points] Solve the following differential equations:

(a) (10 Points)

$$\frac{dy}{dx} = e^{y-x} \cdot \sec y \cdot (1+x^2)$$

with initial condition: $y(0) = 0$.

(b) (15 Points)

$$\frac{d^2y}{dx^2} - 10 \frac{dy}{dx} + 21y = 3 \sin x$$

4. [25 Points]

Given that

$$f_n(x) = \frac{nx}{e^{nx^2}}$$

is a sequence of functions, where $x \in [0, 1]$, and $n = 1, 2, 3, \dots$

(a) (10 Points) Evaluate

$$\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx$$

and

$$\int_0^1 \lim_{n \rightarrow \infty} f_n(x) dx$$

And explain why

$$\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx \neq \int_0^1 \lim_{n \rightarrow \infty} f_n(x) dx$$

(b) (15 Points) Let

$$f_n(x) = \frac{\sin nx}{n^3}$$

where $n = 1, 2, 3, \dots$

Show that

$$\frac{1}{2} \int_0^{\pi} \left[\sum_{n=1}^{\infty} f_n(x) \right] dx - \sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} = 0$$