

元智大學 102 學年度研究所 碩士班 招生試題卷

系(所)別： 電機工程學系碩
士班

組別： 數位科技組

科目： 工程數學

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⊗ 不可使用電子計算機

1. (16%) Events A, B, C are independent. $P(A)=0.8$, $P(B)=0.3$, $P(C)=0.1$.
 - (a) What is the probability that one of the three events happens? (4%)
 - (b) What is the probability that two of the three events happens? (4%)
 - (c) What is the probability that no events happen? (4%)
 - (d) Find $P(A \cup B)$. (4%)

2. (24%) Let X be a number uniformly distributed in $[1,3]$, $Y=X+X$.
 - (a) What is the CDF (Cumulative Distribution Function) of X? (4%)
 - (b) What is the PDF (Probability Distribution Function) of X? (4%)
 - (c) What is the CDF (Cumulative Distribution Function) of Y? (4%)
 - (d) What is the PDF (Cumulative Distribution Function) of Y? (4%)
 - (e) Find out $P(X < 2.5)$ (4%)
 - (f) Find out $P(Y < 5.5)$ (4%)

3. (10%) Find the general solution of the following differential equation,

$$y' + y \cdot f(x) = f(x).$$

4. (20%) Solve following differential equations,
 - (a) $(1+3x^2 + 4xy)dx + 2dy = 0$.
 - (b) $xy' + (x+1)y = e^{-x}$ with initial value $y(1) = 0$

5. (10%) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

6. (20%) For the linear system $Ax=b$ with

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3 \end{bmatrix}$$

Please find:

- (a) The normal equation (5%)
- (b) The least square solution of the system (5%)
- (c) The projection p of b onto the span of the columns of A (5%)
- (d) The orthogonal projection matrix for the span of the columns of A (5%)