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國立臺灣大學 102 學年度碩士班招生考試試題

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(1) (15%) Show that for any odd prime number p,

$$1^2 \cdot 3^2 \cdot 5^2 \cdots (p-2)^2 \equiv (-1)^{\frac{p+1}{2}} \pmod{p}.$$

(2) (15%) Let a group $G = G_1 \times G_2$ be a direct product of subgroups and let H be a normal subgroup of G such that $H \cap G_i = \{1\}, i = 1, 2$. Prove that H is abelian.

(3) (10%) Let H be a normal subgroup of a finite group G and P be a Sylow p-subgroup of G. Show that $H \cap P$ is a Sylow p-subgroup of H.

(4) (15%) Let the symmetric polynomial $f(x_1, x_2, x_3, x_4, x_5) = \sum_{i,j,k \text{ distinct}} x_i^2 x_j^2 x_k$. Express f in terms of the elementary symmetric polynomials.

(5) (15%) Show that the ideal $\langle x^2 + 2x + 3, 2x^3 + 3x^2 + 5x - 5 \rangle$ in $\mathbb{Z}[x]$ is not a principal ideal.

(6) (15%) Let $k(x, y, z)$ be the function field of three variables over a field k. Show that the degree $[k(x, y, z) : k(xy^2, yz^2, zx^2)]$ is equal to 9.

(7) (15%) Show that the only field automorphism of the real field $\mathbb{R}$ is the identity map.

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