1. (15 分) Consider the following linear programming (LP) model

$$\text{Minimize } \mathbf{cx}$$
$$\text{subject to } \mathbf{Ax} \le \mathbf{b}$$
$$\mathbf{x} \ge \mathbf{0}$$

where $\mathbf{A}$ is the coefficient matrix, $\mathbf{x}$ is the variable vector (all variables are non-negative), $\mathbf{b}$ is the right-hand-side vector, and $\mathbf{c}$ is the cost vector.　Suppose we know that $\mathbf{x}_1$ and $\mathbf{x}_2$ are both optimal solutions to this LP model, that is, $\mathbf{x}_1 \ne \mathbf{x}_2$ and $\mathbf{cx}_1 = \mathbf{cx}_2$ is the lowest value for $\mathbf{cx}$.

Is it possible that the above LP model also has a third optimal solution (以上 LP 是否可能具有第三個最佳解？), that is, is it possible that there exists another vector $\mathbf{x}_3$ such that $\mathbf{x}_1 \ne \mathbf{x}_3, \mathbf{x}_2 \ne \mathbf{x}_3$, and $\mathbf{cx}_1 = \mathbf{cx}_2 = \mathbf{cx}_3$?　Please answer "yes" or "no" and give a <u>reason</u> to support your answer.　You can illustrate your reasons with numerical examples, but your reasons cannot be specific numerical examples (可用實際例子來<u>輔助說明</u>理由，但理由不可以只是特殊例子).

2. (15 分) We are writing an airplane scheduling problem as an LP model.　For any flight $j$ (第 $j$ 航班), let $x_j$ be the time at which it will leave airport X ($x_j$ 是航班 $j$ 之起飛時間) and $t_j$ be the duration of the flight ($t_j$ 是航班 $j$ 之飛行時間).　Note that $x_j$ is a decision variable and $t_j$ is a parameter.　Assume that for safety reasons, before a flight arrives at its destination, no other flights can leave airport X (假設因特殊安全考量，在一航班未抵達其目的地之前，其他在機場 X 之航班必須在地面等候).

Given the above safety reasons, we will have "$x_1 - x_2 \ge t_2$ or $x_2 - x_1 \ge t_1$" as constraints for flights 1 and 2 (this is because <u>either</u> flight 1 will leave airport X before flight 2, <u>or</u> flight 2 will leave airport X before flight 1).　But this "or" constraint is not linear.

Consider the following two constraints in which $y$ is a binary variable (i.e., $y$ is either 0 or 1) and $M$ is a very, very big number:

$$My + x_1 - x_2 \ge t_2$$
$$\ldots x_2 - x_1 \ldots$$

Please complete the second constraint with $M$, $y$, and $t_1$, so that the finished second constraint is linear and the two constraints have the same effect (請以 $M$、$y$、以及 $t_1$ 來完成第二個限制式，使得完成後的第二個限制式是線性的，而且這兩個限制式具有與「$x_1 - x_2 \ge t_2$ or $x_2 - x_1 \ge t_1$」完全相同的作用).

3. (20 分) For the following (primal) LP model, please find an optimal solution for both the <u>primal</u> and the <u>dual</u> problems.

$$\text{Minimize } 2x_1 + 3x_2 + 4x_3$$
$$\text{subject to } x_1 + 2x_2 + x_3 \ge 3$$
$$2x_1 - x_2 + 3x_3 \ge 4$$
$$x_1, x_2, x_3 \ge 0$$

注意：背面有試題

4. (5 * 2 = 10 分)

(1) Define a Queuing System. How would you apply Queuing Theory to regulate traffic lights?

(2) Define a Markov Chain. How would you apply Markov Chain to predict the market share of competitors?

5. ( 5 * 4 = 20 分)

NCU Car Wash is an automatic operation with a single bay. On a typical morning, cars arrive at a mean rate of nine per hour, with arrivals tending to follow a Poisson distribution. Service time, including manual drying time, is assumed to be exponentially distributed. Past experience suggests that the mean service time should average five minutes.

(1) System utilization =

(2) Average number of cars in line and service =

(3) Average time cars spend waiting in line =

(4) Percent of idle time =

6. (5 + 15 = 20 分)

A car rental agency has three locations in Chungli City: A, B and C. The agency has a group of delivery drivers to serve all three locations.

- Of the calls to location A, 30% are delivered in area A, 30% are delivered in B, and 40% are delivered in C.
- Of the calls to location B, 40% are delivered in area A, 40% are delivered in B, and 20% are delivered in C.
- Of the calls to location C, 50% are delivered in area A, 30% are delivered in B, and 20% are delivered in C.

(1) What is the transition matrix?

(2) What are the long-term probabilities of serving those three locations?

注意：背面有試題