

中原大學 102 學年度 碩士班 入學考試

102/3/2 13:30 ~ 15:00 電機工程學系電子電路組

誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目：工程數學(主考範圍：線性代數、微分方程) (共 1 頁第 1 頁)

可使用計算機，惟僅限不具可程式及多重記憶者 不可使用計算機

In all of the problems, derivatives y' and y'' are done with respect to variable x :

1. Explain the following terms (名詞解釋):

- (a) (5%) dimension of a vector space (b) (5%) linearly independent
(c) (5%) null space (d) (5%) rank of a matrix

2. (10%) Let V be a finite-dimensional inner product space and V has an orthonormal basis

$\beta = \{x_1, x_2, x_3, \dots, x_n\}$. If x is any vector in V , prove that x can be written as

$$x = \sum_{k=1}^n (x, x_k) \cdot x_k,$$

where (x, x_k) denotes inner product of x and x_k .

3. (10%) A square matrix \mathbf{M} is called skew-symmetric if $\mathbf{M}^T = -\mathbf{M}$, where \mathbf{M}^T denotes transpose of \mathbf{M} . Prove that the set of all $n \times n$ skew-symmetric matrices is a subspace.

4. (10%) Find all of the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 4 & 0 \\ 2 & -4 \end{bmatrix}$.

5. (a) (10%) Show that $F = e^{x+y}$ is an integrating factor of $e^{-y} dx + e^{-x}(-e^{-y} + 1) dy = 0$.

(b) (10%) Find general solution of $e^{-y} dx + e^{-x}(-e^{-y} + 1) dy = 0$.

6. (a) (10%) Find general solution of

$$y'' + 4y = 0.$$

(b) (10%) Solve the initial value problem

$$y'' + 4y = 16 \cos(2x), \quad y(0) = 0, \quad y'(0) = 0.$$

7. (10%) Use the Laplace transform to find solution of

$$y'' + 2y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = -3.$$