中原大學 102 學年度 **碩士班** 入學考試

3 月 2 日 10:00~11:30　　資訊工程學系　　誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目：計算機數學　　（共 2 頁 第 1 頁）

□可使用計算機，惟僅限不具可程式及多重記憶者　　☑不可使用計算機

1. Determine whether each of the following statements is True or False. (10%)
   (每題答對得 2 分、答錯扣 2 分，最多倒扣至此大題為 0 分止。)
   (1) The null space of a matrix A is the set of all solutions of equation $Ax = 0$.
   (2) Any vector set $\{v_1, \ldots, v_p\}$ in R^n is linearly dependent if $p < n$.
   (3) An m × n matrix A has orthonormal columns if and only if $A^TA = I$.
   (4) Let A be an n × n matrix, A is invertible if and only if $\det(A) = 0$.
   (5) Let A be an n × n matrix, A is invertible if and only if $Ax = 0$ has only the trivial solution.

2. Determine the values of '$\boldsymbol{a}$' and '$\boldsymbol{b}$' such that the system of linear equations

$$\begin{cases} x_1 - 2x_2 + 3x_3 = 9 \\ -x_1 + 3x_2 + ax_3 = -4 \\ 2x_1 - 5x_2 + 2x_3 = b \end{cases}$$

   (1) has no solution. $\boldsymbol{a}, \boldsymbol{b}$ = ? (5%)
   (2) has infinite solutions. $\boldsymbol{a}, \boldsymbol{b}$ = ? (5%)
   (3) has an unique solution. $\boldsymbol{a}, \boldsymbol{b}$ = ? (5%)

3. Given a matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ -1 & 1 & -2 \end{bmatrix}$.

   (1) Find the eigenvalues of A. (5%)
   (2) A^{20} = ? (5%)

4. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ a \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ b \end{bmatrix}$, $v_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \\ c \end{bmatrix}$, $y = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 6 \end{bmatrix}$, and $\{v_1, v_2, v_3\}$ is an orthogonal set.
   (1) Determine the values of $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$. (5%)
   (2) Find the orthogonal projection of $\boldsymbol{y}$ onto span$\{v_1, v_2, v_3\}$. (5%)

5. Find the inverse of following matrix A. (5%)

$$A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix}$$

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6. Determine whether each of the following statements is True or False. (10%)
(每題答對得 2 分、答錯扣 2 分，最多倒扣至此大題為 0 分止。)
(1) If the universe of discourse is the set of real numbers, $\forall x \exists y(xy = 1)$.
(2) $\{x\} \subseteq B - A$ if $A = \{x, y\}$ and $B = \{x, \{x, z\}\}$.
(3) For any integer n, if n is not divisible by 2 or 3, n^2-1 must be divisible by 24.
(4) If $ac \equiv bc \pmod{m}$, then $a \equiv b \pmod{m}$.
(5) There is a tree with degrees $4, 2, 2, 2, 2, 1, 1, 1, 1$.

7. Fill in the blanks in the following statements. (3 points for each, 15%)
- There are __(1)__ functions from A to B if $A = \{x, y\}$ and $B = \{x, \{x, z\}\}$.
- Let $\boldsymbol{S}=\{(1,2),(2,4),(3,1),(4,3)\}$ be a relation on $\{1,2,3,4\}$, then $\boldsymbol{S}^6 =$ __(2)__.
- $3^{565} \bmod 140 =$ __(3)__.
- There are __(4)__ distinct bit strings of length *six* with no *four* consecutive *0*s.
- A forest that consists of *6* trees and *55* vertices must have __(5)__ edges.

8. Write down the recursive definitions of the following sets.
Example: The set of all bit strings of even length. (Let λ be the empty string.)
Recursive definition: (Base case): $\lambda \in S$.
(Recursive step): if $w \in S$, then $00w, 01w, 10w, 11w \in S$.
(1) The set of all bit strings of even length that start with *1*. (5%)
(2) The set of all bit strings that have more *0*s than *1*s. (5%)

9. Derive the closed form of a simple function that generates the terms of an infinite sequence beginning with integers *3, 6, 11, 18, 27, 38, 51, 66, 83, 102...* (5%)

10. Imagine that you have *16* coins, one of which is a lighter counterfeit (偽幣), and a free-beam balance (秤). No scale of weight is marked. To find the counterfeit coin, (at least) how many times of weighing are needed? Explain your answer. (5%)

11. How many non-isomorphic un-rooted trees are there with *four* vertices? Draw these trees. (5%)