中原大學 102 學年度 碩士班 入學考試

102/3/2 15:30 ~ 17:00 工業與系統工程學系乙組

誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目：作業研究 （共 3 頁第 1 頁）

■可使用計算機，惟僅限不具可程式及多重記憶者 □不可使用計算機

（只有答案而沒有推導過程或理由，以零分計）

1. In some company, there are 3 products, G1, G2 and G3, to sell. These three products have to be produced in plant 1(P1) and then packed in plant 2(P2). The price for selling one G1 is 3 dollars; for selling one G2 is 1 dollar; for selling one G3 is 4 dollars. In P1, it needs 2 units of resources to produce one G1; it needs 1 unit of resource to produce one G2; it needs 2 units of resources to produce one G3. In P2, in average, it needs 1 worker to pack one G1; it needs 1 worker to pack one G2; it needs 4 workers to pack one G3. Currently, in each day, we have 12 units of resource available in P1 and 9 workers in P2. The production manager, John, wants to determine the optimal selling plan that how many G1, G2 and G3 should be sold in order to have the maximum daily revenue. John decides to use linear program to determine the optimal selling plan.

(a) (5%)Write the above selling planning problem as a primal linear programming problem. And write down the corresponding dual problem.

(b) (5%)Solve the primal problem using simplex method.

(c) (5%)Referring the final table found in (b) and the constraints in the dual problem, use dual theory and complementary basic solution to show that the optimal solution obtained in (b) is indeed an optimal solution.

(d) (5%)Suppose now we are running the optimal selling plan obtained in (b), what is the maximal price you want to pay to buy one unit of resource 1 from the outside market? Why?

(e) (5%)Suppose now we are running the optimal selling plan obtained in (b), and suppose that the resources in P1 are also the workers there. If John have a new worker recruited from the department of human resource, which plant, P1 or P2, will John assign this new worker to, why?

(f) (10%)Follow (a), suppose now we are running the optimal selling plan obtained in (b), what are the allowable ranges of the prices of this three products, respectively? Which prices are sensitive?

(g) (10%)Follow (a), suppose now we are running the optimal selling plan obtained in (b). If now the efficiencies for producing and packing a G2 have been improved to that it needs in every day, only in average 1/2 units of resources to produce one G2 in P1 ; it needs 1/2 workers to pack one G2 in P2. And the price also increases to 2 dollars for each G2 sold. Is the current optimal production going to be changed (use duality theory to make the conclusion)? If yes, what is the new optimal production plan(use

re-optimization to make the conclusion. If you start the simplex method to solve the new problem from the beginning you will have zero point for (g).)

2. To obtain a license for a job in financial analysis, Kevin, needs to take exams through four stages, 1,2,3 and 4 and Kevin should starts from stage 1. After taking the exam at stage 1, 2, or 3, Kevin may pass and proceed to the next stage ; he may fail and be required to take the exam again; he may fail and quit taking all the remaining exams. For the exam at stage 4, Kevin may pass and finally have the license ; he may fail and be required to take the exam again ; he may fail and quit taking all the remaining exams.

   Define state 0,1, 2, 3, 4 and S be our states. State 1, 2, 3 and 4 represent the stage 1, 2, 3 and 4 of the exam, respectively. State 0 is the state that quitting taking any of the exams and state S is the state that obtaining the license. The transition probabilities among states 1,2,3,4 (in this order) are as follows:

$$\begin{bmatrix} 0.1 & 0.5 & & \\ & 0.1 & 0.5 & \\ & & 0.2 & 0.5 \\ & & & 0.5 \end{bmatrix}$$

And after taking the exam of stage 4, the probability to quit is 0.25 and probability to have the license is 0.25.

(a) (10%)What is the probability that Kevin will end up quitting? What is the probability that Kevin will end up having the license?

(b) (10%)How many tests in average Kevin will take before he quit or have the license?

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3. Mary is running a breakfast cafe, called Mary's café, which mainly selling the fresh home-town-taste sandwich. Mary hires a chef to make sandwiches. In order to provide the sandwich to customer upon their arrivals, Mary has to prepare the inventory of sandwiches in storage area in advance. However, if Mary prepares too many sandwiches in the storage area, some of the sandwiches may become cold (not fresh) before they are taken by the arriving customers. Mary decides to prepare only a small number of sandwiches, called base stock, in storage area. For example, Mary prepares only 4 newly made sandwiches in the storage area. Number "4" is called the base stock level. When a customer arrives to Mary's café finding that there are some sandwiches in the storage area, he will take one and leave and, at the same time, the chef will start to make a new sandwich according to the first-come-first-served basis. If the customer finds that there is no sandwich in storage area, he will go away and never come back. Assume that the customers arrive to Mary's café according to a Poisson process with rate $\lambda = 2$ per minute. The time to prepare a new sandwich by a chef is exponentially distributed with rate $\mu = 1$. Now suppose that Mary decides to set base stock level be 4. Let the state be the number of sandwiches in the storage area.

(a) (5%) Write down the transition rate diagram.
(b) (5%) Find the steady-state probabilities.
(c) (5%) What is the average number of customers per minute that can have the sandwiches?
(d) (10%) What is the time in average that a sandwich stays in the storage before it is taken by a customer?
(e) (5%)What is the utilization of the chef?
(f) (5%)If the price of a sandwich is 80 dollars and the pay for the chef is 200 dollars per hour, what is Mary's revenue per hour?