中原大學 102 學年度 **碩士班** 入學考試

102/3/2 13:30~15:00　　工業與系統工程學系甲組 ；工業與系統工程學系乙組

誠實是我們珍視的美德，我們喜愛「拒絕作弊，堅守正直」的你！

科目： 機率與統計　　　　（共 2 頁第 1 頁）

☒可使用計算機，惟僅限不具可程式及多重記憶者　　☐不可使用計算機

1. A quality engineer is interested in the relation between the daily average temperature (X) and the number of defective products manufactured in the same day (Y). Specifically, this quality engineer wants to know whether X and Y are positively correlated, negatively correlated, or independent. Observations of (X, Y) are collected for 30 working days. Let (x_1, y_1), (x_2, y_2),…, (x_{30}, y_{30}) denote the 30 paired observations. How would you use these observations to determine the relation between X and Y (positively correlated, negatively correlated, or independent)? Please be specific. (15 points)

2. Given two estimators $\hat{\theta}_1$ and $\hat{\theta}_2$ for an unknown constant θ, how to determine which estimator is better? Please define "better" first. (15 points)

3. The pH of water samples from a specific lake is a random variable X with probability density function

$$f(x)=\begin{cases} 0.25(x-4) & \text{if } 4\le x<6 \\ 0.5 & \text{if } 6\le x\le 7 \\ 0 & \text{else} \end{cases}$$

(a) Compute the mean E(X) and V(X). (14 points)

(b) How often would you expect to see a pH measurement about 6.5? (6 points)

4. Customers arrive at a store at a Poisson process with rate 10 persons/hour. Define the following random variables:

X_1 = number of customers that arrive during tomorrow morning from 9am to noon,
X_2 = number of customers that arrive during tomorrow morning from 9am to noon if no customer arrives during this morning,
X_3 = time (in hours) until the first customer arrives if the store opens at 9am; and
X_4 = time (in hours) until the 5th customer arrives if the store opens at 9am.

Describe the distributions of X_1, X_2, X_3, and X_4, including the distribution name and the associated distribution parameter values. Be specific. (20 points)

5. Suppose that the weight (in kilograms) of a 20 year-old man is a normal random variable with mean 68 and standard deviation 15.

(a) What percentage of 20-year-old men have weight over 85 kilograms? (10 points)

(b) For the 20-year-old men with weight over 85 kilograms, what percentage of them have weight over 100 kilograms? (10 points)

6. A certain component is critical to the operation of an electrical system and must be replaced immediately upon failure. Suppose that the lifetime of this type of component has an expected value equal to 15 days and standard deviation equal to 5 days. If there are 50 such components on hand, what is the probability that these 50 components are enough for the system to continually operate for the next 800 days? (Hint: Use the central limit theorem.) (10 points)

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Standard Normal Distribution Function $\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{y^2}{2}}dy$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998