

# 中原大學 102 學年度 碩士班 入學考試

102/3/2 13:30 ~ 15:00 應用數學系統計組  
應用數學系統計組(在職生)

誠實是我們珍視的美德，  
我們喜愛「拒絕作弊，堅守正直」的你！

科目： 機率

(共 1 頁第 1 頁)

可使用計算機，惟僅限不具可程式及多重記憶者

不可使用計算機

1. If the moment generating function of  $X$  is  $M(t) = \frac{e^t}{2 - e^t}, t < \ln 2$ . Find p.m.f. of  $X$ . (10%)
2. (a) Let  $P(A \cup B) = 0.76, P(A \cup B') = 0.87$ , and  $P(B') = 1 - P(B)$ . Find  $P(A) = ?$  (10%)  
(b) Let  $P(B') = 1 - p(B), P(A) = \frac{1}{3}, P(B) = \frac{1}{2}$ , and  $P(A \cap B) = \frac{1}{5}$ . Find  $P(A|B') = ?$  (10%)
3. Let  $(X, Y)$  has a bivariate normal distribution with parameters  $\mu_X = 3, \mu_Y = 2, \sigma_X = 2, \sigma_Y = 1$  and  $\rho = 0.6$ . Find the distribution of  $X + Y$ . (20%)
4. Let  $X$  be a random variable of the mixed type having the distribution function
$$F(x) = \begin{cases} 0, & x < 0, \\ \frac{x^2}{4}, & 0 \leq x < 1, \\ \frac{x+1}{4}, & 1 \leq x < 2, \\ 1, & 2 \leq x. \end{cases}$$
(a) Find the mean and the variance of  $X$ .  
(b) Find  $P(\frac{1}{4} < X < 1), P(X = 1), P(X = \frac{1}{2}),$  and  $P(\frac{1}{2} \leq X < 2)$ . (20%)
5. If the moment generating function of a random variable  $W$  is  $M(t) = (1 - 3t)^{-5}$ , find the p.d.f., mean, and variance of  $W$ . (10%)
6. Let  $X_1, X_2, \dots, X_n$  be a random sample from a uniform distribution on  $[0, \theta]$ .  
Let  $\hat{\theta} = \frac{n^3 + 3n + 1}{n^3} \max\{X_1, X_2, \dots, X_n\}$ . Find the limiting distribution of  $n(\hat{\theta} - \theta)$  as  $n \rightarrow \infty$ . (10%)
7. Let  $X_1, X_2, X_3$  be a random sample of size  $n = 3$  from the exponential distribution with p.d.f.  $f(x) = e^{-x}, 0 < x < \infty$ . Find  $P(\max\{X_1, X_2, X_3\} \leq 1)$ . (10%)