中原大學 102 學年度 **碩士班** 入學考試

102/3/2 15:30 ~ 17:00 應用數學系數學組
應用數學系數學組(在職生)

誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目：線性代數　　　　（共 1 頁第 1 頁）

☐可使用計算機，惟僅限不具可程式及多重記憶者　　☑不可使用計算機

[1]. Label the following statements as true or false:

(a) $A=\begin{pmatrix}1 & 1\\ 2 & 2\end{pmatrix}$ is an invertible matrix.

(b) $A,B\in M_n(\mathbb{R})$, then $AB=BA$.

(c) $A\in M_n(\mathbb{R})$, A is not invertible, then A is not diagonalizable.

(d) $T:V(\boldsymbol{F})\to V(\boldsymbol{F})$ is a linear transformation from vector space $V(\boldsymbol{F})$ *to itself* and $\dim V(\boldsymbol{F})=n$, then

T is diagonalizable ⇔ For each order basis β of $V(\boldsymbol{F})$, $[T]_\beta$ is similar to a $n\times n$ diagonal matrix.

(20%)

[2]. Let $A=\begin{pmatrix}1 & 2 & 3\\ 2 & 5 & -3\\ 1 & 0 & 8\end{pmatrix}$, show that A is invertible and find its inverse matrix A^{-1}. (20%)

[3]. Let $S=\{v_1,\dots,v_k\}\subseteq V(\boldsymbol{F})$ (a vector space over $\boldsymbol{F}$). Suppose that S is linearly independent and $v\in V(\boldsymbol{F})$. Prove that

$v\notin \operatorname{span}\{v_1,\dots,v_k\}\Leftrightarrow v\cup S=\{v,v_1,\dots,v_k\}$ is linearly independent. (20%)

[4]. Let $A=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$ and $\alpha=\{\binom{1}{0},\binom{0}{1}\}$, $\beta=\{\binom{1}{0},\binom{1}{1}\}$ be the order basis of $\mathbb{R}^2$. Suppose that $T_A:\mathbb{R}^2\to\mathbb{R}^2$ satisfies $T_A(x)=Ax,\forall x\in\mathbb{R}^2$. Find $Q\in M_2(\mathbb{R})$ such that $[T_A]_\beta=Q[T_A]_\alpha Q^{-1}$. (20%)

[5]. Let $A=\begin{pmatrix}0 & -2 & -3\\ 1 & 3 & 3\\ 0 & 0 & 1\end{pmatrix}_{3\times 3}$, find $A^{50}=?$ (20%)