中原大學 102 學年度 **碩士班** 入學考試

102/3/2 13:30 ~ 15:00 應用數學系數學組
應用數學系數學組(在職生)

誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目： 高等微積分 （共 1 頁第 1 頁）

□可使用計算機，惟僅限不具可程式及多重記憶者 ■不可使用計算機

I. (40 分) Determine each of the following statements is true or false. Explain briefly your answers.

(1) If $f:(-1,1)\to\mathbb{R}$ is differentiable at 0, then

$$\lim_{h\to 0}\frac{f(h)-f(-h)}{h}=f'(0).$$

(2) If both A and B compact sets in $\mathbb{R}^k$, then the set $A+B=\{x+y : x\in A \text{ and } y\in B\}$ is also compact.

(3) Suppose $f:(a-1,a+1)\to\mathbb{R}$ and $g:(A-1,A+1)\to\mathbb{R}$ satisfy:

$$\lim_{x\to a} f(x)=A \text{ and } \lim_{y\to A} g(y)=B.$$

Then $\lim_{x\to a} g(f(x))=B$.

(4) If $n\in\mathbb{N}$ and $f(x)=x^n$, then f is uniformly continuous on $[0,1]$.

(5) If $n\in\mathbb{N}$ and $f_n(x)=x^n$, then the sequence $\{f_n\}$ converges uniformly on $[0,1]$.

II. (12 分) Suppose that $T:\mathbb{R}^n\to\mathbb{R}^m$ is a linear transformation. Show that T is uniformly continuous on $\mathbb{R}^n$.

III. (12 分) Suppose that $f:[-2,2]\to\mathbb{R}$ is a continuous function. Prove that

$$\lim_{h\to 0}\int_{-1}^{1}\left|f(x+h)-f(x)\right|dx=0.$$

IV. Let $f:\mathbb{R}^2\to\mathbb{R}$ be defined by

$$f(x,y)=\sqrt{|xy|}.$$

(1) (8 分) Find the values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the origin $(0,0)$.

(2) (12 分) Is f differentiable at $(0,0)$?

V. (10 分) Suppose that $\{a_n\}$ is a sequence in $\mathbb{C}$ and that $\sum_{n=1}^{\infty}|a_n|^2$ converges. Prove that the series $\sum_{n=1}^{\infty}\frac{a_n}{n}$ converges absolutely.

VI. (6 分) State each of the following well-known theorem.

(1) Heine-Borel Theorem

(2) Weierstrass M-Test