

# 元智大學 102 學年度研究所 碩士班 招生試題卷

系(所)別： 電機工程學系碩士班

組別： 控制工程組

科目： 工程數學

用紙第 | 頁共 | 頁

●不可使用電子計算機

1. The complex Fourier series of a function  $f(x)$  defined on the interval  $(-p, p)$  is

given by  $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\pi x/p}$ . Show that the coefficients  $c_n$  can be obtained as

$$c_n = \frac{1}{2p} \int_{-p}^p f(x) e^{-in\pi x/p} dx, \quad n = 0, \pm 1, \pm 2, \dots \quad (10\%)$$

2. Given  $f(x) = \begin{cases} 0, & -1/2 \leq x \leq -1/4 \\ 1, & -1/4 < x < 1/4 \\ 0, & 1/4 \leq x < 1/2 \end{cases}$  on the interval  $(-1/2, 1/2)$  and  $f(x+1) = f(x)$

for all  $x$ . Find the complex Fourier series  $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in2\pi x}$  and plot  $(n\omega_0, |c_n|)$

in the range  $-6 \leq n \leq 6$ , where  $\omega_0 = 2\pi$ . (20%)

3. A random variable  $X$  has a probability density function given by

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}. \text{ Find the expected value of } g(X) = 1 + 2X + 3X^2. \quad (20\%)$$

4. Find the general solution of

(a)  $3y^4 - 1 + 12xy^3 y' = 0$  (10%)

(b)  $y'' + 4y = x + 2e^{-2x}$  (10%)

5. Solve the initial value problems.

(a)  $y'' + 2y' + 2y = \delta(t-3); y(0) = y'(0) = 0$  (10%)

(b)  $y'' + 4y = f(t); y(0) = y'(0) = 0$  in which  $f(t) = \begin{cases} 0 & \text{for } t < 3 \\ t & \text{for } t \geq 3 \end{cases}$  (10%)

6. Find the general solution of

$$\mathbf{X}' = \begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix} \mathbf{X}. \quad (10\%)$$

