

元智大學 102 學年度研究所 碩士班 招生試題卷

系(所)別：通訊工程學系碩士班 組別：通訊組 科目：工程數學 用紙第 / 頁共 2 頁

●不可使用電子計算機

1. (8%) (Probability Theory) With your own words, please:
  - (a) Define *Event space*. (2%)
  - (b) Define *Random variable*. (3%)
  - (c) Explain *Bayes' theorem*. (3%)
2. (6%) Consider the experiment where a fair die is rolled twice. Verify that the event of getting a "1" the first time the die is rolled and the event of getting a "2" the second time are independent.
3. (8%) Give two CDFs (cumulative distribution functions) to show that "Two random variables with the same first and second moments can have different distributions".
4. (12%) The number of hits at a Web in any 10-second interval is a Poisson random variable with variance 5.
  - (a) What is the probability that there will be no hits in a 2-second interval? (6%)
  - (b) What is the probability that there are no more than two hits in an interval of one second? (6%)
5. (16%) Suppose  $Y$  is a continuous random variable uniformly distributed over  $[-1, 4]$  and  $Z = Y^2$ .
  - (a) Find the CDF of  $Z$ . (8%)
  - (b) Find the second moment of  $Z$ . (8%)
6. (8%) Please show that
$$\det(I + AA^H) = \det(I + A^H A)$$
where  $I$  is an  $n \times n$  identity matrix,  $A$  is an  $n \times n$  square matrix, and 'det' is the operation of taking the determinant.

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7. (15%) Please find the inversion of matrix  $A$  using three methods that you have learned.

$$A = \begin{bmatrix} a_{11} & b_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

where  $a_{11}$ ,  $a_{21}$ ,  $a_{22}$ , and  $b_{12}$  are constant values with  $a_{11} \neq 0$ , and  $|A| \neq 0$ .

8. (15%) Please answer the following questions:

- (a) What's a Hermitian matrix? Given an example. (5%)  
(b) What are the characteristics of the eigenvalues and eigenvectors of a Hermitian matrix?

Based on your example of a Hermitian matrix in (a), please confirm that the eigenvalues and eigenvectors of that Hermitian matrix do have those characteristics. (10%)

9. (12%) Please use the following column vectors to construct a set of orthonormal vectors.

$$x_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$