元智大學 102 學年度研究所 碩士班 招生試題卷

系(所)別：管理學院商學碩士班　組別：財務金融碩士學程　科目：微積分　用紙第 1 頁共 1 頁

●可使用現行『國家考試電子計算器規格標準』規定第一類之計算機

1. (18%) Find the limit of the following functions.

   (a) $\lim_{x\to\infty}\frac{4x}{\sqrt{2x^2+10}}$ (6%)

   (b) $\lim_{x\to\infty}\ln\left(1-\frac{3}{2x}\right)^{2x}$ (6%)

   (c) $\lim_{x\to\infty}\left[x\ln\left(1+2x^{-1}\right)\right]^x$ (6%)

2. (12%) Evaluate the definite integral $\int_0^1\int_y^1 y^2e^{x^2}dxdy$.

3. (12%) Evaluate $\frac{d}{dx}\left[\int_{3x}^{x^2}e^{\frac{-t^2}{2}}dt\right]$.

4. (12%) Let $y=\ln x$ and the probability density function of random variable y is $f(y)=\frac{1}{\sqrt{2\pi}\sigma}e^{\frac{-(y-\mu)^2}{2\sigma^2}}$, please find the expected value of random variable x.

5. (12%) Find the Taylor expansion of the function $f(x)=\ln\frac{1-x}{1+x}$ for $|x|<1$.

6. (14%) Assume a non-constant differentiable function f defined on the set of positive numbers such that $f(x/y)=f(x)-f(y)$, for all $x>0$ and $y>0$. Please show that

   (a) $f(xy)=f(x)+f(y)$. (7%)

   (b) $f'(x)=f'(1)/x$. (7%)

7. (20%) The Black-Scholes formula of an European call option is $C(S,X,\sigma,r,T)=SN(d_1)-Xe^{-rT}N(d_2)$, where $N(.)$ is the standardized Normal distribution, $d_1=\frac{\ln\left(\frac{S}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$, $d_2=d_1-\sigma\sqrt{T}$, and X,σ,r,T are constants. Please derive the Delta $(\frac{\partial C}{\partial S})$ and Gamma $(\frac{\partial^2 C}{\partial S^2})$.