元智大學 102 學年度研究所 **碩士班** 招生試題卷

系(所)別：	資訊工程學系碩士班	組別：	不分組	科目：	離散數學	用紙第 1 頁共 1 頁

●不可使用電子計算機

Notation:

$\mathbb{Z}^+$	The set of **positive integers**
$\mathbb{N}$	The set of **natural numbers** (including 0)
$(G \circ F)(x)$	The **composition** of two functions $G(x)$ and $F(x)$
$\gcd(a,b)$	The **greatest common divisor** of two positive integers a and b
$\mathrm{lcm}(a,b)$	The **least common multiple** of two positive integers a and b

一、填充題（每格 5 分，共 55 分）

1. Let $F: \mathbb{N} \to \mathbb{Z}^+$, $F(x) = \frac{d}{dx}G(x)$, and $G: \mathbb{N} \to \mathbb{Z}^+$, $G(x) = 3x^2 + 5x + 1$.

- **A.** Is $F(x)$ a **one-to-one** (or **injective**) function? _______ (yes or no)
- **B.** Is $F(x)$ an **onto** (or **surjective**) function? _______ (yes or no)
- **C.** A tight (as good as possible) lower (**big-Ω**) bound of $F(x)$ is _______.
- **D.** A tight (as good as possible) upper (**big-O**) bound of $(G \circ F)(x)$ is _______.

2. Let K_n denotes the **complete undirected** graph with total n **vertices** ($n \in \mathbb{Z}^+$).

- **A.** Please draw K_6.
- **B.** By removing appropriate **edges** from K_n, one can obtain a **spanning tree** of K_n. The number of different edges that should be removed is _______.
- **C.** In order to connect K_n and K_m ($m \in \mathbb{Z}^+$) into K_{n+m}, the number of different **edges** that should be additionally drawn is _______.

3. For $a, b, c, d \in \mathbb{Z}^+$, it is known that $\mathbf{lcm}(a,b) = a$, $\mathbf{lcm}(b,c) = b$, and $\mathbf{gcd}(c,d) = d$.

- **A.** $\mathbf{lcm}(c,d) =$_______.
- **B.** $\mathbf{gcd}(a,d) =$_______.

4. Consider a **complete n-ary** tree T of **height h** ($n, h \in \mathbb{Z}^+$, $n \geq 2$). For $l \in \mathbb{Z}^+$, $l \leq h$,

- **A.** The number of **ancestors** of a vertex in T at **level l** is _______.
- **B.** The number of **descendants** of a vertex in T at **level l** is _______.

二、問答題（每題 15 分，共 45 分）

1. For $a, b, c, m \in \mathbb{Z}^+$ and $m > 1$, prove that If $a^2 \not\equiv bc \pmod{m}$, then $(a \not\equiv b \pmod{m}) \vee (a \not\equiv c \pmod{m})$. Please explain your answer in detail.
[**Hint:** If a direct proof is difficult, try other proof techniques, such as **contraposition**.]

2. Consider n distinct vertices. If all the n vertices must be applied to construct a graph, how many different **simple undirected graphs** can be obtained? Please explain your answer in detail.
[**Hint**: This problem is related to **combinations without repetition** and **permutations with repetition**.]

3. For a weighted (undirected) graph $G = (V, E)$, the **maximum spanning tree** $T = (V', E')$ of G is defined as the spanning tree of G whose **sum of edge weights** is the maximum among all the spanning trees of G. Please briefly describe an algorithm (with **pseudo-code**) to find T.
[**Hint**: This problem is similar to finding the **minimum spanning tree** of G.]